

Utility Seeking in Complex Social Systems: An Applied Longitudinal Network Study on Command and Control

Joshua Lospinoso¹, Ian McCulloh^{1&2}, and Kathleen M. Carley²

Abstract. Humans are autonomous, intelligent, and adaptive agents. By adopting social network analysis techniques, we submit a framework for the study of dynamic networks and demonstrate the use of actor-oriented specifications in longitudinal networks. Through the use of a unique command and control dataset from experiments run at the US Military Academy, we illustrate the power of testing hypotheses on actor utility profiles. We frame static, covariate factors onto communication networks, and find that statistical hypothesis testing indicates edge networks truly motivate soldiers to seek information, collaborate, and modify the social network around them into more comfortable configurations of triad closure and edge reciprocity, when compared to hierarchical networks: a finding with profound implications to the study of complex, adaptive social systems.

1 INTRODUCTION

Multi-agent simulation is rapidly emerging as a popular tool for understanding complex social and organizational structures. Historically, these models have been either very simple, or have contained few agents due to issues of computational complexity. As the power of computers continues to increase rapidly, more complex multi-agent simulation models are needed. Social network analysis has become equally popular for understanding social and organizational structures. This paper applies methods in longitudinal social network analysis to multi-agent simulation.

Human organizations and social groups are composed of individuals. The individuals can be related in a number of different ways: friendship, trust, ethnicity, shared ideology, shared goals, and more. Some of these relationships are important in understanding the behaviour and actions of the organization or social group. Other relationships are unimportant. Furthermore, some relationships affect others, creating very complex dynamic behaviour.

Multi-agent simulation is used to model individual agents that can act, interact, and learn. The agents exist in an environment where their interaction is constrained by their position in various social networks defined by the aforementioned relationships among others. Group behaviour emerges as a result of the complex interaction between agents.

Understanding network structure is very important for modelling social groups and organizations in a realistic manner. For example, Valente [1] was interested in modelling the diffusion of contraceptive innovations in the Cameroon. He found that real-world adoption rates did not follow simulation

models when the network relationships were ignored. An individual's decision to adopt an innovation is highly dependent on the decisions of adjacent individuals in a social network. Assumptions of random mixing of individuals, therefore, generate inaccurate adoption rates since trust and friendship networks are important factors. When the simulation accurately models the underlying social networks of people in the Cameroon, more accurate diffusion models are obtained. For a more thorough review of the diffusion of innovations, see Valente [1].

Understanding social networks is not only important for modelling diffusion processes. Social networks are important for modelling any social group or organization involving humans. Multi-agent simulation modellers should be familiar with important theories in social network analysis that govern relationships between individual agents. Incorporating some of these theories into simulation models will contribute to more realistic models.

It is also important to be able to identify what social theories are applicable to certain problems and situations. Relationships that may be important in one context may be unimportant in another. Social network analysts are able to statistically test for the significance of various social theories in longitudinal network data. Equipped with significant theories governing network formation in empirical data, the multi-agent simulation modeller can include these factors in their simulation, thereby creating more realistic agent interactions.

This paper will present a novel approach to multi-agent simulation and demonstrate it on a real-world network data set. Longitudinal network data is collected in a natural experiment focused on studying shared situational awareness and communication. An actor-oriented model [2] is fit to the data to determine significant social theories contributing to network dynamics. These theories can then be incorporated in a multi-agent simulation model to create more accurate organizational behaviour.

The paper is organized as follows. First, we describe a theory of network dynamics used in social network analysis. Next, we describe the concept of network utility. In Section 4 we describe network data collected from a natural experiment conducted at the U.S. Military Academy. Section 5 describes a longitudinal analysis of that data, with the results presented in Section 6. In Section 7, we highlight implications for multi-agent simulation modellers and provide directions for future work.

2 NETWORK DYNAMICS

Network dynamics is a term used in social network analysis to describe the behaviour of networks over time [3,4,5]. Social network analysts have been conducting research in this area for quite some time [2,6,7,8,9,10,11,12,13,14,15]. There are four

¹ Network Science Center, United States Military Academy, West Point, NY 10996. Email: {Joshua.lospinoso, ai6873}@usma.edu.

² Center for Computational Analysis of Social and Organizational Systems (CASOS), Carnegie Mellon University, Pittsburgh, PA 15213. Email: {imccullo, Kathleen.carley}@cs.cmu.edu.

behaviours that can occur in a network over time: *Stability*, *Evolution*, *Random Change*, and *Mutation*.

Network *Stability* occurs when the underlying relationships that connect agents in a network remain the same over time [15]. The observed data may contain error. Some relationships may not be observed, while some observed connections may be inadvertent and no relationship exists. Consider email communication. An agent may communicate with some friends every day, others sporadically, and they may even accidentally email someone they do not know by hitting the wrong name in a distribution list or replying to all in an email. While the observed networks may fluctuate from day to day, the underlying relationships remain unchanged. They have reached a dynamic equilibrium for at least the short term.

Network *Evolution* occurs when agent interaction over time changes the underlying relationships [3]. Furthermore, evolution assumes that there is some underlying stochastic process that causes change over time. There are two leading approaches for modelling network evolution. One general class of approach is to use Markov chains [16,17,18,19,20,21,22]. Under this approach, the network transitions from one network state to the next over time. The future state of the network is conditioned only on the current time step and not previous time steps. Research has focussed on the structure of the transition matrix that governs the evolution of the networks.

An alternate approach for modelling network evolution is multi-agent simulation [3,23,24,25,26,27]. Under this approach, agent based models are created in which agents interact according to some established social theory. Interactions allow the agents to change in some important way that may affect future interaction.

Random Change in a network occurs when the future behaviour of the network is independent of the current state [5]. In other words, the agent interaction is affected by something external to the network. For example, an Army platoon may evolve as individual agents interact and communicate. When that same Army platoon comes under attack by the enemy, there is something fundamentally different about their relationships. There is not anything inherent in the individual agent interactions that could have predicted the change in network behaviour as a result of the enemy attack.

It is also possible that a random change could initiate network evolution [5]. We call this type of behaviour a *Mutation*. In our Army example, it is possible that under the stress of enemy combat an individual agent displays remarkable courage or cowardice. This individual behaviour may improve or remove the status of an agent. Other agents in the network may respond differently to agent based on their actions during the random change.

One possible explanation of network dynamics is agent-driven optimization. Agents in a network attempt to optimize their utility subject to various costs and constraints. Under this concept, stability can be viewed as an equilibrium surrounding some local optima. Evolution can be viewed as the network converging on some new dynamic equilibrium. Random change is still exogenous to the network and changes the state of agents in the network. If this change results in some other local optima, then the network reaches some new stability states. Otherwise, the network experiences mutation as the network converges to a new equilibrium. This concept of agent-driven optimization is

further explored in this paper as an approach for modelling complex adaptive social systems.

3 NETWORK UTILITY

The concept of actor-driven models for network evolution was proposed by Snijders [2,28]. Several applications of this model have been presented [29,30,31]. Snijders' concept of actor-driven models views a network from the perspective of individual agents. Each agent can control the set of outgoing links to other agents in the network. His seminal assumption is that actors perform myopic stochastic optimization in continuous time. These changes are Markovian and depend on network structure, attributes, and observed covariates.

Social network analysts use Snijders' actor-driven model to determine what pre-defined social factors are important in describing the evolution of empirical social network data. Snijders [2] defines 11 potential objective functions that have some sociological meaning:

1. The *density effect* is defined by the number of links an agent has to other agents in the network.
2. The *reciprocity effect* is defined by the number of links to other agents that are reciprocated, in that when an agent links to a target agent, that target also links back to the original agent.
3. The *transitivity effect* is defined by the number of transitive patterns among an agent's connections. A transitive pattern occurs when two of an agent's connections are connected themselves. This is also known as a transitive triplet. Transitivity follows the logic that two agents are more likely to know each other if they have a common friend.
4. The *balance effect* is defined by the similarity of outgoing links between an agent's connections. This theory is driven by the idea that there are positive and negative links and an agent is uncomfortable having both relations simultaneously. In other words the enemy of my friend should be my enemy and the friend of my friend should be my friend. If I am friends with my enemy's friend, I will feel uncomfortable. This effect is highly correlated with the density effect and transitivity effect. If both are included in a model a correction for the correlation between effects should be included.
5. The *number of geodesic distances of two effect* is defined by the number of other agents that an agent is indirectly connected to through an intermediary agent.
6. The *popularity effect* is defined as the number of links an agent has coming from other agents in the network.
7. The *activity effect* is defined as the number of other agents that can be reached by an agent in two steps.
8. The *main link effect* is a covariate effect for links in the network. The other objective functions might be weighted by certain relationships. For example, a link to an agent of high prestige or rank might be more valuable than a link to an agent with equivalent status.
9. The *related popularity effect* is a covariate effect for agents in the network. This is defined for an agent, i , as the sum of the popularity effect of all other agents connected to agent i .
10. The *related activity effect* is a covariate effect for agents in the network. This is defined for an agent, i , as the sum

of the activity effect of all other agents connected to agent i .

11. The *related dissimilarity effect* is a covariate effect for agents in the network. This is defined as the sum of the differences in some important attribute between an agent and its' direct connections.

Agents in a network can also experience constraints as well as objectives. Agents can be constrained in the number of links that they can maintain to other agents in the network. This constraint models cognitive limitations on individuals. A person is not capable of maintaining meaningful relationships with hundreds of people. Other constraints may be imposed on the agents in the network. Snijders does not consider constraints in his model to simplify computation. When estimating the effects, the density effect often has a negative coefficient. This is interpreted as an observed constraint on node degree. See Snijders [2] for a more thorough explanation. Our aim is to present considerations in multi-agent simulation based on social network analysis and not to generate a comprehensive model.

Under a network utility model, an agent will change its outgoing links in such a way as to increase its overall utility, which is equivalent to optimizing its objective function. It is important to note that the list of objective functions are suggestions and are non-exhaustive. When tested against empirical data, only a subset of the objective functions may be found to be significant. Undoubtedly, an analyst could consider other important social factors. Therefore, when using these objective functions in a multi-agent simulation, the modeller should use some intuition in determining important effects. Ideally, a modeller could record empirical data, use Snijders' actor-driven approach to determine significant objective function effects as his approach was intended, and then use those effects in a multi-agent simulation to make inference on the future behaviour of the network.

It is important to point out differences between network utility and classic game theory. Common applications of game theory intend to focus on trading scarce resources. The network utility approach does not consider the transfer of resources, rather agents attempt to optimize their position in their social network. This approach may not be common in multi-agent simulation, but it is supported in the social sciences.

4 DATA

Parity Communications in collaboration with the Higgins Trust Framework and the Social Physics project constructed the ELICIT software package. Installed on client computers, the software serves as the platform for studying organizational efficiency and effectiveness. The four phase experiment entails an introduction, practice round, a one hour exercise, and a wrap up. During both the practice round and the actual exercise, thirty four subjects are randomly assigned to one of two organizations: a typical hierarchically arrayed organization (C2) and a control-free, self-organizing organization (E). These two organizations operate independently for the duration of the exercises.

The goal of the organization is to identify a terrorist attack based on bits of information distributed around the organization. After ten minutes of the one hour experiment, all of the correct information has been issued to the organization. Among the correct bits of information, or factoids, are also distributed false factoids. Each agent receives four factoids, and they must

corroborate within the organization to come up with the correct arrangement of who, what, where, and when. The C2 group is comprised of a squad leader, four team leaders, and twelve team members. Communications among these agents are restricted to the following graph in Figure 1:

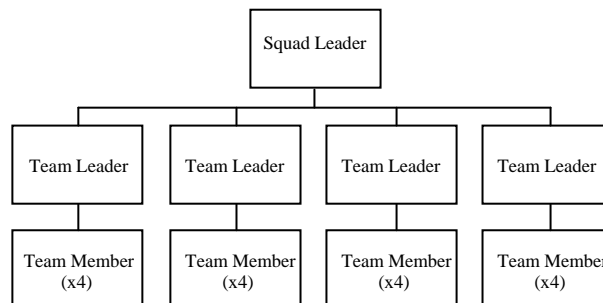


Figure 1. C2 Communications Hierarchy

Each team is dedicated to identifying one key element of the terrorist attack: who, what, where, and when.

The E group is comprised of seventeen agents with full communication capability across the organization. There are no defined teams, but the goal remains the same: positively identify the terrorist attack. All agents have the ability to post their information on their organization's website. Within the E group, this website is global to the organization. The C2 group has separate websites for each echelon (four teams and one squad site). The hierarchy in Figure 1 describes where each agent can post information. Agents can also share information with other individual agents. Once an agent believes that it knows any number of correct factoids, it can report its belief through the "identify" function to its immediate superior in the C2 group or to the entire network in the E group.

Data was collected on two iterations of ELICIT experiments conducted at West Point. During one iteration, the cadets were allowed to communicate within an edge-network configuration. In the other, the cadets were required to adhere to a strict hierarchy. Other than these systemic restrictions, the two iterations were run identically for an actual test run of two hours.

The human participants in this experiment were all cadets at the U.S. Military Academy between the ages of 17 and 23. The experiment was approved for ethics and safety by the West Point Institutional Review Board. All participants received a briefing on the experiment, consented to participate, and had the option to leave the experiment at any time without any adverse impacts. The investigators conducting the experiment were not in the participants' military chain of command, so no undue influence was exerted in this experiment.

5 METHOD

We use the social network software package SIENA [32] which implements an actor-oriented network model [2] to analyze data from two iterations of the ELICIT experiment. Adjacency matrices were constructed to reflect the structure of communication networks over time. These are un-weighted (dichotomous), directed, and non-reflexive square matrices. We must define time intervals in which to discretize or bin the data. Following the guidelines set out by Steglich and Snijders [33],

we chose five bins. Each edge e_{ijt} was assigned a positive value (of one) if one of two conditions was met: cadet i sent cadet j information during time bin t , or cadet i posted information on a team website sometime between the start of the experiment and time t which cadet j retrieved during time t .

Next, we defined covariates. This step is crucial and warrants special attention when conducting an actor-oriented model specification under the SIENA framework. Covariates are empirically derived values which are infused directly into four main objective functions (effects 8-11 above) and provide compelling parameter estimates which can potentially gain critical insight into important aspects of sociological systems. In the case of the ELICIT data, we identify two main link effect covariates corresponding to leadership and location. The *leadership-link effect* is modelled with dependence-style network. The leadership network consists of time-invariant relationships of who was in charge of whom. Note that the leadership network was completely empty for the edge-organization case, because there were no formally defined leadership roles. The statistically significant parameter estimates of the leadership-link effect indicate that formal leadership roles may play a significant part in driving agent behaviour. With low- or even negative--parameter estimates, agents in the network are averse to forming links with formal leaders. The *location-link effect* models geographical proximity. Within the ELICIT framework, geographic distance may play a significant role within the hierarchical network, since geographical locations coincide with team placements. It would seem to also be an important covariate for the agents in the edge network, since agents within the same geographical region post to the same website and are most likely to gain information from this site. The statistically significant parameter estimates of the *location-link effect* indicate a strong affinity or aversion across both the edge and hierarchical networks on the basis of team cohesion (whether enforced or not).

In addition to main link effect covariates defined on relationships between agents, we also defined a covariate for the information an agent possesses. As time progresses in the experiment, agents gain bits of information. Once an agent believes that the information is true, they will privately publish their belief to the ELICIT server, where the belief can be recorded by the experiment administrators. This is a time varying effect. We use the related popularity effect (number 9 above) to model this effect. Statistically significant parameter estimates of the *information effect* indicate that agents with more information attract more communication from other agents in the network.

We also modelled the density effect, the reciprocity effect, and the transitivity effect (effects 1-3 above), because they are commonly used in the literature. We elected to omit other objective functions to prevent over specification of the model. See Steglich and Snijders for a more comprehensive review [33].

6 RESULTS

In order to estimate the parameters of both the edge and hierarchical treatments simultaneously, we compiled both adjacency matrices and covariates into large matrices with structural holes where appropriate. We conducted estimation procedures within SIENA using default parameters and 1000 iterations of the three-stage Metropolis-Hastings Markov Chain

Monte Carlo. Tables 1 and 2 display the parameter estimates of the E and C2 networks respectively.

Measure	Parameter Estimate (<i>p-val</i>)
Density Effect	-.3693 (.028)
Transitivity Effect	.2054 (.031)
Reciprocity Effect	.1502 (.070)
Location-link Effect	.0513 (.471)
Leadership-link Effect	--
Information Effect	.2146 (.009)

Table 1. Parameter Estimates for Edge Network

Measure	Parameter Estimate (<i>p-val</i>)
Density Effect	-.9976 (.035)
Transitivity Effect	.2007 (.044)
Reciprocity Effect	.0640 (.36)
Location-link Effect	.2632 (.017)
Leadership-link Effect	.1507 (.023)
Information Effect	-.1647 (.019)

Table 2. Parameter Estimates for Hierarchical Network

We estimate six important objective functions to determine what sort of utility profiles are recurrent in each of the networks. After separating out the effects of each of the networks using individual covariate dummy variables, we find that the density effect measure is negative and statistically significant, which corresponds with our intuition that there is some sort of underlying cost to adding edges. Within the edge network, this effect is significantly diminished, which may indicate that agents in the edge network either have more cognitive capacity to form ties or that they are empowered by a lack of formal hierarchical structure. We find that the magnitude of this estimate (nearly -1) compared to the relative size of the other objective functions indicates that there are strong limitations to the cognitive capacity of the agents within the hierarchical network.

Transitivity effect has a strong and statistically significant, positive parameter estimate. Agents in both of these networks tend to close triads, which would confirm our intuition in the hierarchical network, where team members might be expected to close triads within their teams. The estimates are rather stable across the edge/hierarchical treatment, and it would appear that there is little difference between the two utility profiles.

Reciprocity effect has little effect within the hierarchical network, but it has a significant effect on the edge network. Reciprocity tells us how likely one node is to return information to the entity who sent them information. This supports our intuition that in an edge network, relationships are created on the basis of information necessity and all agents must cross-load information. Within the hierarchical network, team-leaders can ask for information and receive information without ever having to inform their teams what is going on; so the edges are not reciprocated (which is why we fail to have statistically significant results under the hierarchical network).

Location-link effect has a statistically significant effect on the parameters for the hierarchical network. This may be a result of location and team membership being highly correlated. When two agents in the hierarchical network are within a team, their team leader tasks them with determining one of the factoids, so it is natural that collaboration here should become important. Within the edge network, there is no statistically significant

estimate for location. What this indicates is that within the edge network, covariates of initial team membership mean little and agents quickly breakout of their location to connect with the other locations and help contribute to their knowledge base.

Leadership-link effect was estimated for the hierarchical network and had a strong, positive estimate. This indicates that the leadership role could explain a large portion of variation in the communication patterns of the hierarchy. It both supports our intuition and supports the notion that leadership within the hierarchy was effective at promoting information sharing up and down the chain.

Information effect parameter estimates differed considerably between the edge and hierarchical treatments. Within the hierarchical network, there was actually a strong, negative correlation between people who had assembled information into some sort of conclusion and others. This means that there is information hoarding going on in the hierarchical network; the leadership is hoarding the information. Within the edge network, people who have assembled information seem to attract many edges. We cannot establish causality directly from this estimate (i.e. it could be that the entity has information because he is highly interconnected, or that he is interconnected because he has information), but it is certain that information sharing within the network is a largely significant behavioural engine.

There are some striking differences about the behaviour of these two networks. First, information sharing and collaboration occurs much more within the edge network, while leadership seems to drive much of the behaviour in the hierarchical network. Agents in the edge network tended develop sharing relationships much more than in the hierarchical network as evidenced by the high reciprocity and triad closure in the edge network. Finally, it appears that edge network agents had fewer constraints on collaboration *en masse* as indicated by the magnitude of their density effect estimates.

7 CONCLUSIONS & FUTURE WORK

Defence agencies of the future will increasingly rely on an understanding of complex systems. From understanding the asymmetrical nature (non-hierarchical) of armed adversaries to engineering net-centric systems that maximize efficiency and effectiveness, researchers have and will continue to benefit from empirical studies of complex systems--whether social, physical, or biological [33,34,35]. For a thorough review on this active area of research, the reader is referred to Alberts [34].

We utilized an actor-oriented specification of a complex social system as opposed to an aggregated, holistic assessment of the system, and as a result we were able to dig into the underlying behavioural mechanics of the network and truly understand what is driving the autonomous, intelligent behaviour of the cadets in the study. We now understand that soldiers within *net-centric* edge networks do collaborate across geographic and formal boundaries as expected, but more importantly--their behaviour is *driven* by the need to accumulate knowledge and settle into comfortable social patterns (like triad consensus, reciprocity, etc.).

Beyond contributing to sociological literature and the defence industry's understanding of net-centric operations and systems, this paper has introduced actor-oriented models in social network analysis which identify statistically significant utility seeking behaviour within empirical data. The study of complex,

adaptive systems can benefit from this empirical framework by permitting the investigator a deep look into the underlying mechanics that drive network structure. Enabled with these tools, there is a considerable array of future directions that investigators can pursue to enrich our understanding of complex systems.

Parameter estimates from an actor-oriented specification as outlined in this paper can be used to drive a multi-agent simulation. Moreover, the approach laid out in this paper allows a modeller to use empirical data to determine factors driving agent interaction within a simulation. Building simulation based on statistically significant findings within empirical data is an important aspect of model verification.

This approach requires that multi-agent simulation frameworks are capable of modelling significant utility seeking behaviour. It is important to note that functions driving agent behaviour may differ among differing applications. In the ELICIT example, different objective functions were significant for the edge and hierarchical networks, even given highly homogeneous sets of agents. This implies that there is no one model that fits all applications.

An example of a flexible multi-agent simulation is *Construct*, which is a multi-agent simulation developed by the Center for Computational Analysis of Social and Organizational Systems [24,25]. *Construct* models agent interaction by assigning probabilities of link formation between agents at each time step. The probability of link formation is determined by a weighted function of homophily, socio-demographics, and proximity. Throughout the simulation, agents interact, share knowledge, and change in various attributes as a result of interaction with other agents. Within the framework laid out in this paper, homophily is equivalent to transitivity, reciprocity, balance, and the information effect. Socio-demographics are equivalent to the number of geodesics of two effect, the popularity effect and the activity effect as well as some covariate effects. The proximity is equivalent to a main-link effect. Other effects can be incorporated into the *Construct* model as well. While a detailed explanation of *Construct* is beyond the scope of this paper, we point out that it is an example of a multi-agent simulation framework that can be used to simulate empirically observed network data. The statistically significant parameter estimates of the actor-oriented model can be used to provide weights to the functions that determine the probability of link formation between agents. In this manner, the predictive power of the multi-agent simulation is enhanced due to it being closely tied to empirical data. Future work should explore the ramifications of resolving utility profiles into probability profiles.

An empirically grounded multi-agent simulation also contributes to better understanding network dynamics. This paper serves to unify competing approaches to modelling network evolution. Future work may explore opportunities to introduce random change into the simulated networks. Realistic simulation of networks allows investigators to explore network dynamics by introducing various forms of evolutionary and random change at known points in time and observing their behaviour. This is necessary for exploring networks over time.

The approach presented in this paper is still limited in several ways. The list of objective function effects outlined in Section 3 is not exhaustive. There are likely other important utility seeking functions governing agent interaction. Some effects are highly correlated and including too many effects may lead to

over specified or degenerate models. Future work may investigate additional objective functions for actor-oriented models.

Hopefully, multi-agent system researchers will be motivated to apply an actor-oriented approach to empirical network data. The determination of statistically significant utility seeking behaviour in networks offers us a deep, complexity-preserving insight into the underlying behaviour of social systems. Whether the information is used at face value to draw inference on sociological, physical, and biological phenomena, or utilized as an intermediary to simulation analysis, empirical analysis of the utility seeking behaviour characterizing complex networks around us promises to deepen our understanding of them.

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REFERENCES

- [1] T. Valente. Network Models and Methods for Studying the Diffusion of Innovations. In: *Models and Methods in Social Network Analysis*. P. Carrington, J. Scott, S. Wasserman (Eds.) Cambridge Press (2007).
- [2] T.A.B. Snijders Models for longitudinal network data. In: *Models and Methods in Social Network Analysis*. P. Carrington, J. Scott, S. Wasserman (Eds.) Cambridge Press (2007).
- [3] P. Doreian and F. Stokman. *Evolution of Social Networks*. Gordon and Breach, Amsterdam, (1997).
- [4] I. McCulloh and K.M.Carley. Dynamic Network Change Detection. In: *Proceedings of the 26th Army Science Conference*. U.S.Army, Orlando, FL, (2008)
- [5] I. McCulloh and K.M.Carley. *Detecting Change in Longitudinal Social Networks*. Unpublished manuscript, (2008).
- [6] S.Sampson. *Crisis in a cloister*. Ph.D. Thesis. Cornell University, Ithaca, (1969).
- [7] T. Newcomb. *The Acquaintance Process*. Holt, Rinehart and Winston, New York, (1961).
- [8] A. Romney, A.K. Quantitative models, science and cumulative knowledge. *Quantitative Anthropology*, 1: 153-223 (1989).
- [9] A. Sanil, D. Banks, and K.M. Carley. Models for evolving fixed node networks: Model fitting and model testing. *Social Networks*, 17. (1995).
- [10] T.A.B. Snijders. Testing for change in a digraph at two time points. *Social Networks*, 12: 539-573 (1990)
- [11] O. Frank. Statistical analysis of change in networks. *Statistica Neerlandica* 45: 283-293. (1991)
- [12] M. Huisman, and T.A.B. Snijders. Statistical analysis of longitudinal network data with changing composition. *Sociological Methods and Research*, 32:253-287 (2003).
- [13] J. Johnson, J. Boster, and L. Palinkas. Social roles and the evolution of networks in extreme and isolated environments. *Mathematical Sociology*, 27: 89-121 (2003).
- [14] I. McCulloh, G. Garcia, K. Tardieu, J. MacGibon, H. Dye, K. Moores, J. Graham, and D. Horn. *IkeNet: Social network analysis of e-mail traffic in the Eisenhower Leadership Development Program*. (Technical Report, No. 1218). U.S. Army Research Institute for the Behavioral and Social Sciences. Arlington, VA (2007a).
- [15] I. McCulloh, J. Lospinoso, and K.M. Carley. Social Network Probability Mechanics. In: *Proceedings of the World Scientific Engineering Academy and Society 12th International Conference on Applied Mathematics*, WSEAS, Cairo, Egypt. (2007b).
- [16] P. Holland, and S. Leinhardt. A dynamic model for social networks. *Mathematical Sociology*, 5:5-20 (1977).
- [17] S. Wasserman. *Stochastic Models for Directed Graphs*. Ph.D. dissertation, Harvard University, Department of Statistics, Cambridge, MA (1977).
- [18] S. Wasserman. A stochastic model for directed graphs with transition rates determined by reciprocity. In: *Sociological Methodology*. K.F. Schuessler (Ed.) Jossey-Bass, San Francisco CA, 392-412 (1979).
- [19] S. Wasserman. Analyzing social networks as stochastic processes. *American Statistical Association*, 75: 280-294 (1980).
- [20] R. Leenders. Models for network dynamics: a Markovian framework. *Mathematical Sociology*, 20: 1-21 (1995).
- [21] T. A. B. Snijders, and M.A.J. van Duijn. Simulation for Statistical Inference in Dynamic Network Models. In: *Simulating Social Phenomena*. R. Conte, R. Hegselmann, and P. Tera (Eds.) Springer, Berlin, Germany, 493-512 (1997).
- [22] T.A.B. Snijders., The statistical evaluation of social network dynamics. In: *Sociological Methodology*. M.E. Sobel, and M.P. Becker (Eds.) Basil Blackwell, Boston, MA, 361-395 (2001).
- [23] P. Doreian. On the evolution of group and network structures. II. Structures within structure. *Social Networks*, 8: 33-64 (1983).
- [24] K.M. Carley. A theory of group stability. *American Sociology Review*, 56(3): 331-354 (1991).
- [25] K.M. Carley. Group Stability: A Socio-Cognitive Approach. *Advances in Group Processes*, 7: 1-44 (1990).
- [26] K.M. Carley. Communication Technologies and Their Effect on Cultural Homogeneity, Consensus, and the Diffusion of New Ideas. *Sociological Perspectives*, 38(4): 547-571 (1995).
- [27] K.M. Carley. On the evolution of social and organizational networks. *Research in the Sociology of Organizations*, 16: 3-30 (1999).
- [28] Snijders, T.A.B. (1996). Stochastic actor-oriented models for network change. *Journal of Mathematical Sociology*, 21, 149-172.
- [29] G. Van de Bunt, M.A.J. van Duijn, and T.A.B. Snijders. Friendship networks through time: An actor-oriented statistical network model. *Computational and Mathematical Organization Theory*, 5: 167-192 (1999).
- [30] W. de Nooy. The dynamics of artistic prestige. *Poetics*, 30:147-167 (2002).
- [31] M.A.J. van Duijn, E.P.H. Zeggelink, J.M. Huisman, F.N. Stokman, and F.W. Wasseur. Evolution of sociology freshmen into a friendship network. *Mathematical Sociology*, 27: 153-191 (2003).
- [32] T.A.B. Snijders, C.E.G. Steglich, M. Schweinberger, and M. Huisman. *Manual for SIENA version 3.1*. University of Groningen: ICS / Department of Sociology; University of Oxford: Department of Statistics (2007).
- [33] C.E.G. Steglich, T.A.B. Snijders, and P. West. Applying SIENA: An Illustrative Analysis of the Coevolution of Adolescents' Friendship Networks, Taste in Music, and Alcohol Consumption. *Methodology*, 2: 48-56 (2006).
- [34] Alberts, D. *Power to the Edge*. Washington, DC: CISSP, (2002).
- [35] J.A. Lospinoso. Utility Maximizing Networks. In: *Proceedings of the 2008 International Conference on Information and Knowledge Engineering*, Las Vegas, NV (2008)
- [36] J.A. Lospinoso. The ELICIT Experiment: Eliciting Organizational Effectiveness and Efficiency under Shared Belief. In: *Proceedings of the 12th International Command and Control Research and Technology Symposium*, CCRP, Washington D.C. (2007)