Labour Supply Dynamics and Psychological Fatigue

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Abstract. This paper models psychological fatigue in the context of a labour supply problem with state-dependent utility, where individuals accumulate psychological fatigue as a direct consequence of hours worked. The process is driven by two core processes: (i) the trade-off between job demands and job resources, which dictates how important hours worked are for the dynamics of fatigue, and how responsive those dynamics are to a change in hours worked; and (ii) the recovery from fatigue after supplying hours worked. The model shows that individuals who are more resilient with regards to fatigue will work more and become more mentally exhausted. Furthermore, greater sensitivity to fatigue, independently of recuperation, will lead workers to supply less labour and accumulate less fatigue over time. finally, individuals will begin by working longer hours when psychological fatigue is low, before decreasing their hours worked as they become more mentally exhausted over time.

1 INTRODUCTION

In recent years, reports from international organisations have started to look at the mental well-being of individuals and have been reporting that workers face significant psychological stress coming from the workplace, reporting that up to 50% of the working population of the OECD will experience some degree of mental ill-health at some point during their career ([19], [20]). This is also confirmed in the psychology literature, where the negative effects of work on the mental health of individuals has been investigated for decades now, with models such as the Job-Demands Resources (JDR) framework, which weighs job demands against job resources and finds that an imbalance in favour of the former leads to job burnout [5]. In recent decades, the share of workers experiencing psychological distress coming from their work environment has been increasing, and yet no effort has been made in economic theory to account for this phenomenon.

From the standpoint of economics, the main effect of mental is that it leads to variations in labour supply, and is positively correlated with employee turnover (see [12], [27] and [6]). Understanding the determinants of labour supply is important for designing taxes, or formulating labour market policies such as welfare programmes and social insurances. This paper argues that formalising an economic framework for mental fatigue is therefore important, as the determinants of hours worked are crucial for understanding the effects of economic policies, and the prevalence of work-related emotional exhaustion in rich countries is substantial, reaching up to 38% in the UK and around 25% in Norway, the Netherlands and Australia (see [21], [10], [13], and [3]).

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The objective of this paper is therefore to establish a mathematically tractable and intuitive framework for studying the dynamics of psychological exhaustion and their impact on labour supply. The existing theories and approaches in the psychology literature offer only limited insight into questions pertinent to economics, such as the effects over time of accumulated fatigue on behaviour and the relative importance of the individual's decision-making process. Thus, the aim of this paper is to formalise these views into a coherent and consistent structure, which can then be used to determine how fatigue accumulates exactly, and what the effects of perceived workload, individuals' sensitivity to fatigue and their natural ability to recover are on individual behaviour. Additionally, the paper contributes to the growing literature in behavioural macroeconomics, by setting the stage for further economic analysis around the concept of fatigue.

The consumer in the model maximises lifetime utility, where labour income is the only source of earnings. There is no savings decision nor any accumulation of human capital. The only source of dynamics comes from the accumulation of fatigue through hours worked. The choice of hours worked therefore creates a trade-off between increasing consumption by increasing labour income receipts, while also increasing psychological exhaustion. The reasoning behind this is that the individual accumulates fatigue, which in turn reduces the individual's well-being by lowering utility. The individual has a given sensitivity to fatigue, which will determine to what extent the latter affects well-being. Fatigue accumulates deterministically over time in hours worked and perceived workload, while the recovery process is exogenous but specific to the individual.

Modelling workload exogenously will capture features of the JDR model. A higher level of perceived workload will be consistent with increasing job demands (or decreasing job resources). This measure of the effect of hours worked on fatigue can also capture the trade-off observed in the Efforts-Rewards Imbalance (ERI) model [23], which predicts that when efforts at work exceed rewards, psychological fatigue will increase faster in hours worked. On the other hand, the recuperation component of fatigue, also modelled exogenously, will follow the logic of the Effort-Recovery (ER) model [17]. In the ER model, efforts required to perform tasks at work lead to the individual feeling progressively more exhausted as a direct reaction to ad hoc work-related demands, while the individual will recover smoothly from this expense of efforts automatically when they are not working, or when demands are sufficiently low. Thus, in keeping with the psychological theory, both workload and recuperation are modelled exogenously.

The model predicts that individuals will begin by supplying a high level of labour supply, before progressively adjusting their hours worked downwards over time as psychological fatigue increases. Growth in labour supply in the model will only be observed if hours worked are above a certain threshold value, which will be dependent on the underlying wage dynamics. Optimal behaviour predicts a compensatory motive for workers whereby the impact of a welfare loss due to fatigue will be compensated by an increase in hours worked. Concentrating on the new features introduced in the model, individuals who are more fatigue-resistant will work longer hours and accumulate more fatigue in the long-run, while more fatigue-sensitive individuals will also work fewer hours and accumulate less mental fatigue in the long-run. Greater sensitivity of the accumulation of fatigue to hours worked will lead individuals to work longer hours but accumulate less fatigue in the long-run. More resilient individuals, who recover faster, will increase hours worked but also fatigue in the long run, due to the increased capacity to cope with emotional exhaustion. A higher workload will lead individuals to lower labour supply, but the effect on fatigue will depend on the elasticity of utility in consumption, if the individual's utility does not react strongly to changes in consumption then fatigue will be lower in the long run, while if the individual's utility reacts strongly to changes in consumption then fatigue will be higher in the long run. Finally, individuals will begin by working longer hours when they are psychologically healthy, before reducing their labour supply as their mental state progressively suffers from the accumulated psychological exhaustion.

2 THE MODEL

2.1 Mental fatigue and well-being

Time is continuous and indexed by $\tau \in T := [t, \infty)$, consumers derive utility from consuming goods at time τ, denoted $c(\tau) \in R_+$, while experiencing disutility from psychological fatigue at time τ, measured by $g(\tau) \in R_+$. Fatigue is defined on the set of non-negative reals, the reasoning behind this being that 'negative' fatigue has no definition. Individuals can either be psychologically and mentally exhausted or not, and if so, the question then is to what degree they are exhausted. Emotional exhaustion is positively correlated with higher levels of anxiety , sleep disturbance and cognitive impairments [22], but also with lower life satisfaction and depression [9]. These negative effects on individual well-being are captured below in (1). The individual chooses consumption in period τ, while psychological fatigue is a stock variable that affects her well-being directly. The instantaneous utility function is of the form

$$u(c(\tau), g(\tau)), \tag{1}$$

and has the following restrictions. The function $u : R_+^2 \longrightarrow R$ is C^2 in both of its arguments, and is strictly increasing concave in c

$$u_{cc} < 0 < u_c, \tag{2}$$

and decreasing concave in g

$$u_{gg}, u_g \leq 0. \tag{3}$$

Concavity of u in g captures the fact that as psychological fatigue increases, its effects on individual well-being become increasingly more severe. Furthermore, u is additively separable in c and g, meaning that the cross-partial derivatives are equal to zero,

$$u_{cg} = u_{gc} = 0. \tag{4}$$

We can find further support for this modelling choice by looking at the concept of allostatic load in biology and cognitive sciences. The notion of allostasis describes a process through which an organism corrects imbalances through physiological or behavioural changes in order to reach homeostasis [25], which we can relate to individuals following a dynamic path to equilibrium in economics. Relatedly, allostatic load refers to the accumulated cost of allostasis [16]. We can here see that the accumulation of fatigue acts as a signal to the individual to correct their behaviour. The accumulated allostatic load corresponds to the stock or level of psychological fatigue felt by the individual, and affects her negatively by reducing her overall well-being. A similar model has been advanced for stress, called the allostatic model of stress by McEwen [15], where he describes stress as arising from a prolonged exposure of the body to allostatic overload. In the model presented here, emotional exhaustion accumulates over time as a reaction to work-related tasks, and as it accumulates its strain on individual welfare increases at an increasing rate. Thus I model only the direct effect of fatigue on the individual, and abstract from indirect effects such as cognitive impairments, which may affect productivity.

Intertemporal preferences are given by the following objective function giving lifetime utility as the discounted sum of instantaneous utility in (1), where $\rho \in R_{++}$ is the time preference rate,

$$U(t) = \int_t^\infty e^{-\rho[\tau - t]} u(c(\tau), g(\tau)) \, d\tau. \tag{5}$$

Maximising over an infinite time horizon allows us to prevent the individual from solving the model backwards and to obtain solutions that are independent of any terminal condition. I assume that the individual cannot save or borrow, and consumption is therefore limited to equal her labour income, with wage $w(\tau) \in R_{++}$ and productivity $\theta \in R_+$,

$$c(\tau) = \theta h(\tau) w(\tau). \tag{6}$$

The individual chooses consumption $c(\tau)$ in every period τ, which determines how much labour $h(\tau) \in [0, 1]$ to supply, taken to represent the total share of time endowment the individual can choose from in any period. The time endowment is normalised to the closed unit interval, representing the share of total time available spent working. However, we can see directly from (6) that the problem is equivalent to selecting how much of her time endowment to work, i.e. choosing $h(\tau)$, which will automatically determine the consumption level, given the productivity-adjusted wage, $\theta w(\tau)$.

2.2 Dynamics of exhaustion

This section describes the accumulation of mental fatigue via a first-order autonomous differential equation, where fatigue accumulates deterministically as a function of hours worked and the current stock of fatigue in period τ. This approach follows the tradition of the JDR and ER models, which state that hours worked increase emotional exhaustion through job demands and job resources, and that the accumulation of fatigue will be slower if the individual can recover, respectively. High job demands and low job resources lead to an increase in emotional exhaustion as the individual is psychologically overtaxed in order to cope with the demanding aspects of work and the potential lack of necessary resources. If the individual has sufficient resources to deal with her job demands, then the accumulation of fatigue in hours worked is reduced, that is the slope of the function becomes less steep. I use the following expression to model fatigue accumulation,

$$\dot{g}(\tau) = f(h(\tau), g(\tau)), \tag{7}$$

where I impose the following restrictions on $\dot{g}(\tau)$. The function $f : [0,1] \times R_+ \longrightarrow R$ is C^2 in both of its arguments, and is strictly increasing convex in h

$$0 < f_{hh}, f_h, \tag{8}$$

and strictly non-increasing in g

$$f_g \leq 0. \tag{9}$$

Furthermore, f is additively separable in h and g, meaning that the cross-partial derivatives are also equal to zero

$$f_{hg} = f_{gh} = 0. \tag{10}$$

Thus, I only make the following assumptions regarding the partial derivatives of the fatigue accumulation function. Firstly, hours worked have a strictly non-negative marginal effect on the growth rate of fatigue. This allows us to rule out situations where working somehow reduces the negative effect of psychological fatigue, which could include situations such as individuals alleviating their stress or even recuperating through working, an unlikely but not impossible scenario. Furthermore, hours worked are assumed to have an increasingly negative impact on the growth rate of fatigue, by convexity. This implies that, ceteris paribus, the more hours an individual works, the faster her mental fatigue will accumulate, meaning that for example working one more hour from 12h/day to 13h/day will have a greater effect on fatigue accumulation than working one more hour from 2h/day to 3h/day. Secondly, fatigue is assumed to have a strictly non-positive effect on $f(h,g)$, meaning that it can either have no marginal effect at all or it can affect the growth rate of fatigue as some form of depreciation, whereby from one moment to the next, the individual naturally recuperates and recovers from fatigue. This follows the logic of the ER model, whereby the individual naturally recovers from fatigue. The recuperation process is modelled exogenously to represent the fact that individuals cannot choose their own degree of resistance to psychological fatigue.

This assumption simplifies the analysis by making the steady-state level g^* stable and unique, the latter holds true, if and only if $f(h,g)$ is strictly monotone in g, while the former holds by the assumptions made on $f(h,g)$, regarding the sign of its derivative with respect to fatigue. Having a unique g^* may appear to be a strong assumption economically, however it would be more surprising psychologically if the individual had multiple optimal levels of fatigue for a given set of parameters. While it would present interesting features of the model, it would be unrealistic to model such a situation. Also, I assume the cross-partial derivatives to be zero, meaning that f_g is not dependent on the level of h, while f_h is not dependent on the level of g. In other words, the marginal effect of hours worked on fatigue does not depend on the level of fatigue, while the marginal effect of fatigue on the accumulation process does not depend on the level of hours worked. That is, I separate the recovery process from the effects of on-the-job demands and resources, where the former describes an individual-level phenomenon and the latter describes the effect of the work environment on the dynamics of emotional exhaustion. Allowing for non-zero cross partial derivatives would imply that the ability of the individual to recover from fatigue would somehow be dependent on her work environment and the amount of hours worked, but also that the effect of job demands and resources would depend on the level of fatigue of the individual as well as on their recovery capabilities. While the latter argument may be defensible, the former seems to be more of a stretch both psychologically and rationally. I thus choose to restrict the function to have zero cross-partial derivatives, in order to be as consistent with established psychological facts as possible.

2.3 Optimal rule for hours worked

The individual maximises discounted lifetime utility using (1), subject to (6), and (7), yielding the equation in (11) (see Appendix for steps), which when combined with the fatigue accumulation function and the budget constraint, yields the reduced form of our system in general form

$$\dot{h}(\tau) = \frac{f_g(\tau) - \rho - \frac{u_g(\tau)}{u_c(\tau)}\frac{f_h(\tau)}{\theta w(\tau)} + \left[1 + \frac{u_{cc}(\tau)}{u_c(\tau)} c(\tau)\right]\frac{\dot{w}(\tau)}{w(\tau)}}{\frac{f_{hh}(\tau)}{f_h(\tau)} - \frac{u_{cc}(\tau)}{u_c(\tau)} w(\tau)}. \tag{11}$$

Focusing on (11) we can see that the only force driving the growth rate of hours worked upward is a compensatory motive determined by the ratio of the marginal utilities, the $\frac{u_g(\tau)}{u_c(\tau)}$ term. This term shows a trade-off between the marginal cost of hours worked and their marginal benefit on the instantaneous utility function, weighed by the ratio of the productivity-adjusted wage $\theta w(\tau)$ to the marginal effect of hours worked on the growth rate of fatigue, $f_h(\tau)$, this is summarised in Lemma 1 below.

Lemma 1 (Compensatory motive) *Individuals will increase their hours worked over time when marginal disutility of fatigue increases in order to compensate for the marginal utility loss, relative to the marginal utility from consumption.*

This term shows that if the marginal disutility of fatigue relative to the marginal utility of consumption increases, the individual will increase her hours worked faster, in order to compensate for the increasing relative marginal disutility. The marginal effect of $u_g(\tau)$ on the growth rate of hours worked is measured by the inverse of the marginal utility of consumption, as well as the relative cost of hours worked on the accumulation of fatigue over the wage, functioning as a real measure of an increase in hours worked on fatigue. We can further see that, in the numerator of (11), the wage rate will have an ambiguous effect on the growth rate of hours worked, as it will have a negative effect in levels, but its growth rate will have either a positive or negative effect depending on the sign of the term in the square brackets. While in the denominator, the wage rate in levels will have a negative effect overall as it increases. Finally, we can see from (11) that ρ and $f_g(\tau)$ will both have a strictly non-positive effect on the growth rate of hours worked. As the individual values the present more, i.e. as ρ increases, they will choose a lower labour supply path, while an increase in the marginal effect of fatigue on its on growth rate, i.e. a greater rate of depreciation of the stock of fatigue or greater recuperation rate, will also put the individual on a lower labour supply path. The remaining equations described by (6) and (7) are as in Section 2 above, and describe the process of accumulation of psychological fatigue and the budget constraint, respectively. Together, these three equations pin down the endogenous variables of the model, namely labour supply h, fatigue g and consumption c. This system of equations completely characterises the model in equilibrium.

We can now establish the condition under which we can observe growth in labour supply over time. This condition is summarised below in Lemma 2.

It is worth nothing that $h^*(\tau)$ in Lemma 2 is not a steady-state because of the growth rate of wages, that does not need to be equal to zero. Thus, if wages change over time, so will the threshold beyond which individuals increase their labour supply. If individuals face an increasing wage profile over time, this threshold will therefore move upward for a high income growth rate, before the absolute wage level starts eclipsing this effect altogether, reducing the labour supply threshold determining growth in hours worked.

Lemma 2 (Labour supply growth) *Individuals will increase their labour supply over time, if and only if labour supply is above a threshold level $h^*(\tau)$ defined by*

$$\rho - f_g(h^*(\tau), g^*(\tau)) = \left[1 + \frac{u_{cc}(c^*(\tau), g^*(\tau))}{u_c(c^*(\tau), g^*(\tau))} c^*(\tau)\right] \frac{\dot{w}(\tau)}{w(\tau)} - \frac{u_g(c^*(\tau), g^*(\tau))}{u_c(c^*(\tau), g^*(\tau))} \frac{f_h(h^*(\tau), g^*(\tau))}{\theta w(\tau)}. \quad (12)$$

2.4 Modelling recovery and workload explicitly

I will now take the system of equations above as described by (6), (7) and (11) and consider a specific functional form for (1), that satisfies the assumptions made and retains a form common in the labour supply literature,

$$u(c(\tau), g(\tau)) = \frac{c(\tau)^{1-\eta} - 1}{1-\eta} - \beta \frac{g(\tau)^{1+\gamma}}{1+\gamma}, \quad (13)$$

where $\eta \in R_{++}$ is the inverse of the intertemporal elasticity of substitution, $\beta \in R_+$ is a weight parameter for the relative importance of the disutility of fatigue in the instantaneous utility function, and $\gamma \in R_+$ is the elasticity of fatigue, and must be strictly non-negative in order to satisfy the assumptions on utility and ensure concavity. Therefore we can see that if $\beta = 0$ in (13) then $u_g(\tau) = 0$, and we return to the more traditional CRRA utility function, by shutting down the direct effect of psychological fatigue on well-being. However, if $\beta > 0$ then $u_g(\tau) < 0$, and thus individuals will experience disutility in fatigue. This modelling choice then allows us to more clearly assess the effects of being fatigue-resistant (low β) or fatigue-sensitive (high β) on individuals. I also use a specific functional form for the accumulation of psychological fatigue in (7),

$$f(h(\tau) g(\tau)) = \phi \frac{h(\tau)^{1+\psi}}{1+\psi} - \delta g(\tau), \quad (14)$$

where the first term on the right-hand side of (14) captures the trade-off between job demands and job resources, and the second term models the recovery process.

The balance between demands and resources is the main mediator for the progression of emotional exhaustion and is determined by the individual's work environment ([5], [2]). The two parameters associated with this trade-off are $\phi \in R_{++}$ and $\psi \in R_+$, where ϕ measures the relative weight of the effects of the (im)balance between demands and resources in the accumulation of fatigue, while ψ measures the elasticity of this process with respect to hours worked. We can think of these parameters as capturing the effects of workload, supervisor support, and work engagement, on the individual. We can see that if this process becomes more responsive to hours worked (higher ψ), then the partial effect of hours worked on fatigue accumulation will decrease, and if $\psi \to \infty$ then the individual's exhaustion dynamics would only depend on the recovery process[2]. If this becomes less responsive however (lower ψ), then as $\psi \to 0$ then the marginal effect of hours worked on fatigue accumulation will converge to the weight of the demands-resources trade-off in (14), i.e. it will become linear in $h(\tau)$ with coefficient ϕ. We should note here that the reason for these parameters to be exogenous to the model is that they pertain to the external work environment of the individual, thus they are not directly affected by the behaviour of the individual, nor are they chosen.

I further assume that the individual has a natural physical and mental ability to recover from and cope with her own psychological exhaustion, and thus being either more resilient (high δ) or more susceptible (low δ) to it, but also by resting between periods of work. Another way to think of this recuperation is to think of the time not invested in labour supply as being used to regularly engage in activities to reduce her level of fatigue, either through exercising, resting, or engaging in social activities in order to mediate the negative effects of fatigue. This process is captured by the parameter $\delta \in R_+$, which we will call the recuperation rate of the individual, and will act as a type of "depreciation" for $g(\tau)$. We can also think of this parameter as signalling the degree of physical fitness of an individual, with more physically fit individuals having a higher capacity to cope with psychological fatigue. This finds further empirical evidence in the literature (see [1] and [4]), where we find that individuals who exercise less have higher levels of fatigue, showing that physical exercise also has a role to play in the recovery process. Here I assume that this depreciation rate or recuperation rate is linear in fatigue and thus, does not depend on the level of fatigue. This assumption of monotonicity of $\dot{g}(\tau)$ in $g(\tau)$ implies that there exists only one steady-state, as discussed in Section 2.2. Relaxing this assumption could allow for multiple steady-states, however the focus of this paper is on the qualitative aspect of recuperation for the individual. If there were multiple fixed-points, say one stable and one unstable, then the only difference to our current setup would be that there would be a range of values for $g(\tau)$ where the system would converge, while beyond a certain threshold it would diverge away from any equilibrium. If this were the case, we would then focus on the requirements for fatigue to be within the range where our system would converge, and return to the analysis we are already doing here, except with the addition of an extra condition on fatigue. The choice of recuperation, or coping, strategies are not modelled here and cannot be endogenously chosen by the individual. The assumption is that the worker can only affect how much labour she supplies while recuperating naturally through non-work activities. Thus I take the recovering strategies as given, and focus solely on the labour supply problem of the individual.

Using (13) and (14) in the system described by (6), (7) and (11), we can use (6) to replace in (11) and rearrange to obtain the following system of equations,

$$\dot{h}(\tau) = \frac{h(\tau)}{\psi+\eta} \left[\beta g(\tau)^{\gamma} \frac{\phi h(\tau)^{\psi+\eta}}{[\theta w(\tau)]^{1-\eta}} - \delta - \rho + [1-\eta] \frac{\dot{w}(\tau)}{w(\tau)}\right] \quad (15)$$

$$\dot{g}(\tau) = \phi \frac{h(\tau)^{1+\psi}}{1+\psi} - \delta g(\tau). \quad (16)$$

We can see the dynamic rule for hours worked with the accumulation of fatigue, pinning down the endogenous variables of the model. These two equations now describe the reduced form of the system, where we can study the properties for existence of the steady-state, understanding how the parameters of the model affect it. Furthermore, we can then use this system to study the phase diagram in $h(\tau)$ and $g(\tau)$, and how labour supply and fatigue evolve over time.

3 DYNAMICS OF THE LABOUR-FATIGUE PROCESS

3.1 Long-run behaviour and psychological resilience

The system described by (15)-(16) finds its steady-state when labour supply and fatigue growth are equal to zero, and with a constant

[2] Since $h(\tau) \in [0,1]$, we have $\lim_{\psi \to \infty} \phi \frac{h(\tau)^{1+\psi}}{1+\psi} - \delta g(\tau) = -\delta g(\tau)$.

wage rate. If the wage rate is allowed to change over time, then the long-run value for hours worked and fatigue will vary over time with income (see Lemma 2). In other words, a steady-state can only be found when the following conditions are met,

$$\dot{h}(\tau) = 0, \dot{g}(\tau) = 0, \dot{w}(\tau) = 0. \tag{17}$$

Our system of two equations in two variables can easily be solved to obtain equilibrium steady-state values for labour supply and fatigue, using the conditions above. These steady-state values for hours worked and mental fatigue are summarised in Proposition 1, and allow us to understand the channels through which we can affect the long-run behaviour of individuals.

Proposition 1 (Long-run equilibrium) *Defining the following term for ease of notation,*

$$\chi \equiv \psi + \eta + \gamma(1+\psi), \tag{18}$$

we can write the steady-state level of hours worked as

$$h^* = \left[\frac{[\theta w]^{1-\eta}}{\beta} \frac{\rho+\delta}{\phi} \left(\frac{\phi/\delta}{1+\psi} \right)^{-\gamma} \right]^{\frac{1}{\chi}}, \tag{19}$$

and the corresponding steady-state level of fatigue as

$$g^* = \left[\frac{[\theta w]^{1-\eta}}{\beta} \frac{\rho+\delta}{\phi} \left(\frac{\phi/\delta}{1+\psi} \right)^{\frac{\eta+\psi}{1+\psi}} \right]^{\frac{1+\psi}{\chi}}. \tag{20}$$

We can see from expressions (19) and (20) that the determinants of hours worked and fatigue in the steady-state will be very similar. Specifically, we can see that the ratio of the perceived workload (ϕ) to the recuperation rate (δ), weighed by the gross elasticity of exhaustion $(1+\psi)$ in hours worked will have different effects depending on whether one looks at hours worked or fatigue. In the case of the former, it will have a negative effect owing to the elasticity of utility in fatigue (γ). While the latter will be pushed upwards due to the elasticity of utility in consumption (η) and that of fatigue accumulation in hours worked (ψ).

In studying Proposition 1, we can decompose the long-run effects of the underlying parameter set into two corollaries.

Corollary 1 (Labour paths)

(i) *Individuals will be put on a higher labour path, if they are more resilient, the accumulation of fatigue becomes more sensitive to hours worked, as well as if they become more patient.*
(ii) *Individuals will be put on a lower labour path, if they become more fatigue-sensitive, the weight of hours worked in the accumulation of fatigue increases, and if utility becomes more sensitive to changes consumption.*
(iii) *Higher labour income will put individuals on a higher labour path if the substitution effect dominates the income effect.*

In the long-run more resilient individuals (higher δ) will work longer hours. This happens via two distinct channels, first through the direct effect being able to recover more from fatigue, allowing the consumer to work longer hours, and secondly indirectly by slowing down the accumulation of fatigue and thus reducing the impact of emotional exhaustion on the individual. This can be achieved by engaging in social activities aimed at relieving stress outside of work, or by improving the quality of the time spent off work in order to recuperate better. Increases in the elasticity of fatigue accumulation in hours worked (higher ψ), leads to an increase in labour supply in the long-run through two channels, indirectly through the accumulation of fatigue, that is as fatigue becomes more responsive to hours worked, it increases faster, leading to an increase in hours worked in equilibrium to compensate. And directly through the marginal effect of hours worked on its own growth rate, that is, as fatigue becomes more sensitive to responsive to hours worked, the individual will increase labour supply faster to offset the corresponding increase in fatigue. On the other hand, a lower long-run value for hours worked, and consequently a lower labour path, are obtained when individuals themselves, rather than fatigue accumulation, become more sensitive to fatigue (higher γ). In this case, individuals experience greater direct disutility from fatigue, i.e. its negative effects become more important, and thus individuals choose to work fewer hours by adjusting their labour path. Similarly, increases in workload (higher ϕ) decrease long-run hours worked, by directly deterring individuals from working, since any increases in work due to a higher workload will be weighed by the gross elasticity of fatigue in the utility function $(1+\gamma)$.

Corollary 2 (Fatigue paths)

(i) *Individuals will be put on a higher fatigue path, if they become more impatient, as well as if they are more resilient.*
(ii) *Individuals will be put on a lower fatigue path, if they are more fatigue-sensitive, individuals become more receptive to exhaustion, the accumulation of fatigue becomes more elastic in labour supply, or if the income effect becomes stronger relative to the substitution effect.*
(iii) *Increases in the productivity-adjusted wage will lead to a higher fatigue path, while increases in the weight of hours worked in the accumulation of fatigue will put individuals on a lower fatigue path, if the substitution effect dominates the income effect.*

From Corollary 2 above, we can see that impatient and resilient individuals will accumulate more fatigue over time, by being placed on a higher fatigue path. Impatient individuals (higher ρ) will value the present more, and thus work more hours to earn more income, in order to consume more, with the side effect of pushing up the accumulation of fatigue simultaneously. On the other hand, resilient individuals (higher δ) will also be put on a higher fatigue path because of their ability to recover better. That is, since these individuals recuperate faster from fatigue instantaneously, they work longer hours and thus increase their fatigue in the long-run. Further, increasing marginal disutility of fatigue for the individual, either for individuals more sensitive to fatigue (higher γ) or for more fatigue-sensitive individuals (higher β), leads to a lower fatigue path in the long-run. This is a simple behavioural reaction to the increasing negative effect of fatigue on the individual. As the individual becomes more sensitive to fatigue, she progressively adjusts her hours worked downwards, also leading to a reduction in fatigue over time. Similarly when the accumulation of fatigue becomes more responsive to hours worked (higher ψ), individuals will once again react by slowing down the accumulation of fatigue through reducing their long-run hours worked. As η increases, that is as the income effect becomes stronger relative to the substitution effect, or as relative risk aversion increases, in a model with uncertainty, long-run fatigue will also decrease. That is, as utility becomes increasingly concave in consumption, marginal utility in consumption will become higher for lower levels of consumption, leading individuals to adjust their hours worked downwards in the long-run, as higher levels of consumption will yield lower marginal utility than before, thus leading to a lower level of long-run fatigue. Lastly, when η is small enough, and the substitution effect dominates over the income effect, two competing effects take place, on the one hand higher wages will lead to higher long-run fatigue, by leading individuals to increase their hours worked and take advantage of their higher income as the relative change in prices between fatigue and consumption changes; while on the other hand, increasing workload (higher ϕ) will lead individuals to reduce their steady-state level of fatigue and put them on a lower fatigue path. This is due to the marginal effect of hours worked on the accumulation of fatigue increasing through ϕ, leading individuals to decrease their hours worked, and thus their fatigue level. Increases in the perceived workload can occur either directly, through increasing work demands or diminishing job resources, but also indirectly, for example if individuals cannot separate their work life from their private life. Failure to psychologically detach themselves from their work can reduce individuals' ability to recover by effectively putting added pressure on themselves about work-related issues in an environment where they should normally be recuperating ([24]).

3.2 Equilibrium path and psychological health

In order to analyse the transitional dynamics of the model, we must look at the two-dimensional system in h and g by taking the derivative of hours worked with respect to psychological fatigue, considering exhaustion as the exogenous variable using the system in (15)-(16),

$$\frac{\partial h(g)}{\partial g} = \frac{\frac{h(g)}{\psi+\eta}\left[\beta g^{\gamma}\frac{\phi h(g)^{\psi+\eta}}{[\theta w(\tau)]^{1-\eta}} - \delta - \rho + [1-\eta]\frac{\dot{w}(\tau)}{w(\tau)}\right]}{\phi\frac{h(g)^{1+\psi}}{1+\psi} - \delta g}. \quad (21)$$

As we can see above in (21), the system is not time invariant so long as $\dot{w}(\tau) \neq 0$. Thus variations in the wage will cause the equilibrium path described by (21) to move over time, and lead individuals to adjust their labour supply accordingly. This pattern will depend heavily on the size of the intertemporal elasticity of substitution (η), and whether it is greater or lower than one. If, on the other hand we assume no wage growth, such that the only changes to income are unanticipated, the system then finds a stable saddle-path to equilibrium (see Appendix for derivations), presented in Figure 1 below, where the equilibrium path is described by the time-invariant version of (21) above, given by

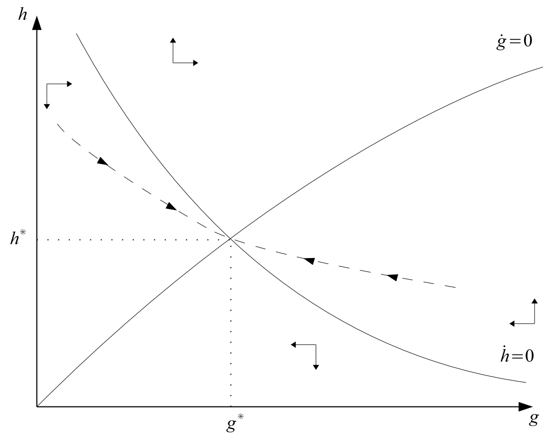
$$\frac{\partial h(g)}{\partial g} = \frac{\frac{h(g)}{\psi+\eta}\left[\beta g^{\gamma}\frac{\phi h(g)^{\psi+\eta}}{[\theta w]^{1-\eta}} - \delta - \rho\right]}{\phi\frac{h(g)^{1+\psi}}{1+\psi} - \delta g}. \quad (22)$$

Furthermore, we can ensure that the individual remains on the saddle-path to equilibrium, so long as the transversality condition (TVC), defined as

$$\lim_{\tau\to\infty} e^{-\rho[\tau-t]}\lambda(\tau)\,g(\tau) = 0, \quad (23)$$

holds, where λ is the associated co-state variable of the model (see Appendix). This condition ensures that fatigue cannot grow too fast compared to its marginal value fixed by the shadow price λ. This condition is equivalent to saying that the individual cannot gain by deviating from the optimal path implied by equation (23) and never returning to it. Thus the TVC, combined with the expression in (22), gives us the optimal path for hours worked and fatigue leading to the maximal level of lifetime utility the individual can achieve.

Figure 1. Dynamics of labour supply and fatigue along the stable saddle-path



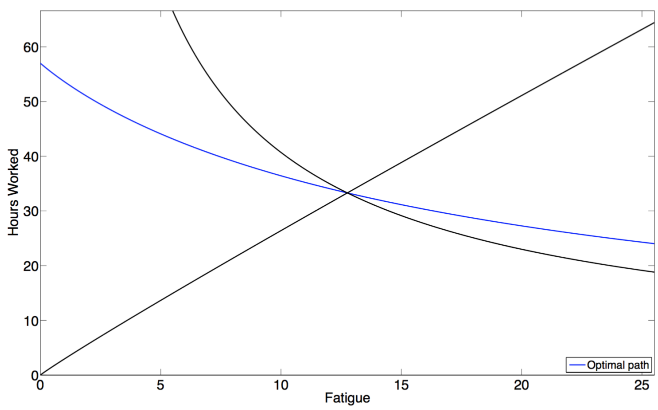
Proposition 2 (Optimal path) *Psychologically healthy individuals will work longer hours and reduce their labour supply as their fatigue increases, while psychologically exhausted individuals will work fewer hours and increase their labour supply as they recover.*

Proposition 2 summarises the main message of Figure 1. When the fatigue level is low, individuals will work a higher number of hours, and progressively reduce their labour supply as mental fatigue starts to accumulate. This behaviour is driven by individuals taking advantage of their low level of exhaustion to work more and thus be able to consume more and maximise their utility. However, as fatigue begins to settle in and take its toll on the individual's psychological state, they will reduce their hours worked as the marginal effect of fatigue on the utility becomes larger compared to the marginal effect of consumption. On the other hand, if the individual fatigue level is high, individuals will work fewer hours, in order to allow themselves to recover from that state of psychological exhaustion, and they will gradually increase their labour supply as their mental health improves, until they can reach their long-run value for hours worked and fatigue, characterised by the steady-states h^* and g^*. Deviations from the optimal path, though prevented by the TVC, would lead individuals either to over work themselves and over accumulate fatigue, or on the contrary to work a sub-optimal amount of hours and never become psychologically exhausted (or recovering too much if coming from the right-hand side of Figure 1), and eventually reduce their hours worked down to zero. Consequently, working the optimal number of hours, as determined by the TVC and the first-order condition, has a stimulating effect on individuals where they remain engaged and choose to work rather than either overworking themselves or losing interest in their work and eventually leaving employment, when $h = 0$. The question now remains to know how long individuals take to reach their steady-state, or an arbitrarily close neighbourhood of their steady-state.

4 QUANTITATIVE RESULTS

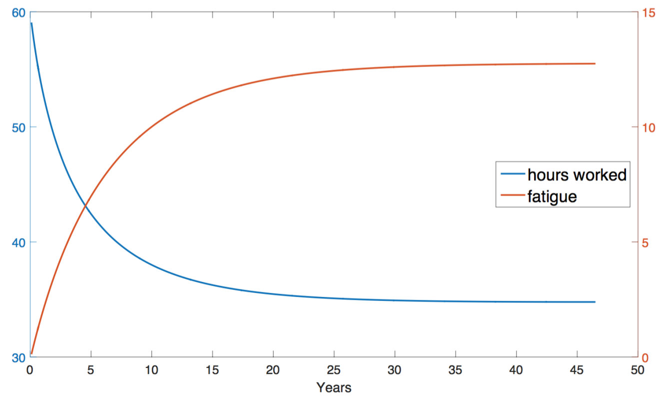
The numerical analysis begins by showing the saddle-path to equilibrium from Figure 1 quantitatively. We therefore look to Figure 2 to show the numerical counterpart to the phase diagram in the previous section. As expected, the optimal path to the steady-state declines monotonically in fatigue, leading to a long-run value of hours worked of around 34h/week, corresponding to the average weekly hours worked in Germany ([18], [26]), and a value for long-run fatigue of around 12.7, corresponding to the average score for individuals in full-time employment [11]. Therefore we can see that the numerical analysis confirms the analytical solution derived previously and shows a substantial shift in hours worked depending on the level of psychological fatigue. If on the other hand the individual begins to the right of the steady-state, where her mental fatigue level is high and hours worked are low, then she will progressively adjust her work hours upward while she recovers and her fatigue reduces. The effects of an unanticipated vacation on the equilibrium path would be to reduce hours worked to zero, while leaving the individual at the same level of fatigue, she would therefore jump to the horizontal axis and progressively move to the left as her fatigue reduces. Eventually, upon returning to work, the worker would jump back immediately on her optimal path, as dictated by the TVC in (23), and she would resume her decline in hours worked and accumulation of fatigue. This strategy of unanticipated time-off can be relevant when the individual finds herself to the right of the steady-state where the level of fatigue is high. Figure 3 below plots the system characterised by (15)-(16)

Figure 2. Hours worked decline as psychological fatigue increases



over time. The time horizon taken is in years, while the left-hand scale shows hours worked per week and the right-hand scale shows the fatigue level. Individuals will begin at time 0 by working close to 60 hours per week when psychological fatigue is close to zero, however rapidly, when facing a constant wage income, hours worked will decline and within 10 years approach 40 hours per week before slowing down and asymptotically converging to the steady state over the remainder of the time horizon. Meanwhile, psychological fatigue will accumulate and converge to its steady state value within 25 years. Thus, as individuals start their working lives with no mental fatigue, they can afford to work longer hours and maximise their labour income, which corresponds exactly to consumption, while the progressive accumulation of the psychological strain on the individual will lead her to reduce her hours worked over time, converging towards her steady-state level. If a sudden unanticipated permanent positive shock to income were to occur, we would observe a discrete jump in the hours profile upwards while the fatigue path would show a kink and an increased steepness at the point where wage increases, eventually leading to higher long-run values for labour supply and mental fatigue. On the other hand, an unanticipated negative permanent shock to wages would have the exact opposite effect an lead to a lower steady-state. Productivity shocks would have the same effects, qualitatively. Thus unanticipated shocks yield only limited insight into behaviour. If the shocks were anticipated, then the paths for labour supply and mental fatigue would adjust slowly over time, either upward in the case of increasing productivity or income, or downward in the case of decreasing productivity or income. Nevertheless, Figure 3 shows that even in the absence of wage dynamics, uncertainty, or human capital, substantial variations in hours worked are still observed in the model once this fatigue accumulation channel is taken into account. This downward trend in hours worked can be related to newer workers who invest too many hours in their job, working overtime at home or on the job, before slowly adjusting to more "normal" or more common levels of labour supply.

Figure 3. Hours and fatigue profiles over time



5 APPLICATIONS

5.1 Behavioural macroeconomics

The model presented here can be adapted to study further questions in economics, such as understanding the determinants of labour supply over the life-cycle, given a non-linear wage profile in finite horizon. In this case the interest would be in understanding to what extent accounting for fatigue in the labour supply process helps explain variations in hours worked late in the working life of individuals at the macro level, a puzzle which persists to this day in the data.

Another extension of the model would be to look at the effects of burnout, the natural consequence of emotional fatigue, on individual behaviour and see how it develops over the life-cycle of the affected individuals. The rate of incidence of job burnout in the OECD is known to be high, and is often claimed to be increasing in recent decades [8]. Even though a more thorough analysis of the data may be needed to obtain accurate figures, it remains that burnout affects real economic variables and should therefore be studied in the context of economics [14].

5.2 Computational modelling

Taking the structure of the model developed in this paper, one could incorporate its components into a more comprehensive framework, following the computational modelling of emotions tradition. To this end we could identify some of the key elements and features of the model. Here the steps through which the individual processes fatigue is given by the following structure, which is closely related to a FAtiMA architecture [7]:

1. perception of environment (e.g. learning about model parameters),
2. knowledge / memory is updated (here implying individual learns her new environment),
3. appraisal (she evaluates impact of environment using the model's structure),
4. optimal state is determined (her long-run values for fatigue and hours worked), and
5. action is taken (level of hours worked is chosen to fit).

6 CONCLUSION

The model predicts that fatigue-resistant as well as more resilient individuals will actually work longer hours and accumulate more psychological fatigue, while more fatigue-sensitive individuals will work fewer hours and accumulate less fatigue. This result, that fatigue-resistant individuals end up accumulating more fatigue, is somewhat surprising and its implications for behaviour as of yet unexplored in the literature. Furthermore, reducing the perceived workload of workers would actually lead to individuals working longer hours, but the effect on fatigue will depend on the intertemporal elasticity of substitution. An interesting caveat of the model is that if the process of fatigue accumulation becomes more sensitive to hours worked, individuals will work longer hours but accumulate less fatigue over time. The model also predicts that individuals will begin their working life by oversupplying labour, taking advantage of their being psychologically healthy at the time, and then progressively reduce their working hours over time as mental exhaustion accumulates and starts negatively affecting their well-being.

We can see therefore that mental fatigue plays a key role in determining hours worked over time, and that further work must now be undertaken in order to understand the effect of assets and human capital accumulation in the presence of fatigue. The presence of savings would allow the individual to partially insure against the negative effects of fatigue and would smooth out consumption, as well as hours worked in the long-run. Modelling the effect of human capital would be a more complex enterprise, as one would need to ascertain the relationship between psychological fatigue and human capital. So far the psychology literature has found only weak evidence pointing to differences in fatigue levels across various education groups, with a tipping point for individuals who have completed primary education and a decreasing prevalence with further years of schooling ([3], [13]). Thus investing in human capital early on could also serve as a form of insurance against future levels of mental exhaustion. Another application could be to consider the effects of a burnout on the individual, which is the natural extension to psychological fatigue, and bears further health risks that are relevant both for health economics and labour economics [14].

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