An Overview of Kinetic Approaches to Study Consensus Formation

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Abstract. This paper discusses some recent results in the study of opinion dynamics and consensus formation in multi-agent systems. While results in these fields are typically obtained by simulation, the adoption of the approach inspired by kinetic theory of gases allows deriving analytic results. More precisely, analytic relationships between the parameters of the model and the asymptotic characteristics of the systems can be analytically determined. Various models inspired from kinetic theory of gases have been studied. For instance, different microscopic phenomena can be investigated, such as compromise and diffusions, and both deterministic and stochastic parameters have been used to model such phenomena. Recently, models inspired from kinetic theory of gas mixtures have also been introduced in the context of opinion dynamics. Such models allow studying multi-agent systems in which different classes of agents coexist. Agents belonging to different classes may be associated with different features, such as the initial distribution of the opinion and the propensity of agents to change their opinions when interacting with other agents. A discussion on the features of different models is presented in this paper.

1 Introduction

This paper discusses analytical approaches to opinion dynamics and consensus formation. Such disciplines are considered important in the fields of multi-agent systems and distributed computing and they apply to various areas, such as robotics, control theory, mathematics, and sociology (e.g., [5, 9]). For this reason, they have been studied using various approaches, among which it is worth recalling those based on graph theory and properties of graph Laplacians [3, 11], on thermodynamics (e.g., [10]), and on Bayesian networks [4]. In this paper we focus on models inspired from kinetic theory of gases, a discipline which studies macroscopic properties of gases, starting from the modeling of microscopic interactions among molecules. According to such models, a parallelism between molecules in gases and agents in multi-agent systems can be defined, so that the kinetic framework can be extended to study the effects of interactions among agents in multi-agent systems.

In this paper we focus on models inspired from kinetic theory of gases, a discipline which studies macroscopic properties of gases, starting from the modeling of microscopic interactions among molecules. Following the ideas behind such models, a parallelism between molecules in gases and agents in multi-agent systems can be defined, so that the kinetic framework can be extended to study the effects of interactions among agents in multi-agent systems. More precisely, we consider multi-agent systems where each agent is associated with a continuous parameter v which represents its opinion. Such parameter can be defined on a closed interval, typically chosen as $I = [-1, 1]$ [7], or on $\mathbb{R}$ [1]. The aim of the considered models is to study, analytically, the temporal evolution of opinion, starting from the details of single interactions among agents. Of course, different interaction rules lead to different macroscopic properties of the system, as it will be discussed in the following. However, the proposed kinetic-based approach allows deriving a general framework which can be used to study different interaction rules and to derive the respective asymptotic opinion distributions, denoted as stationary profiles. The kinetic-inspired approach relies on some assumptions. First, any agent can change its opinion only as a consequence of an interaction with another agent. Moreover, it is assumed that each agent can interact with any other agent in the multi-agent system and that each interaction is binary, namely it involves only two agents [6]. Observe that this last assumption is not restrictive, since interactions involving many agents can be modeled as a set of binary interactions. Finally, time is modeled as a sequence of discrete steps, which do not necessarily have the same duration. Each step corresponds to a single interaction between two agents [8].

This paper is organized as follows. Section 2 presents an overview of different models inspired from kinetic theory, starting from different rules that model the post-interaction opinions as functions of the pre-interaction ones. Section 3 concludes the paper.

2 The Kinetic Formulation of Opinion Dynamics

In this paper, we describe results derived according to different models, obtained by considering different interaction rules among agents. We remark that such rules are inspired by phenomena typically studied by sociologists and their explicit formulation differs from that used to model interactions among molecules in kinetic theory. Despite these differences, the kinetic framework can be successfully adopted to obtain analytic results in the field of opinion dynamics in multi-agent systems. Analytic results are independent from the tools and from the actual values used for simulations. The availability of analytic results also allows the design of systems with desired properties, without having to repeatedly simulate the behaviour of the system with different parameters until the desired properties are obtained. For these reasons, the availability of analytic results should be considered as a key advantage of the proposed framework. In the remaining of this paper, we associate to each agent a scalar parameter v, which represents the agent's opinion. We assume that $v \in I = [-1, 1]$, so that ± 1 represent extremal opinions, while values close to 0 correspond to moderate opinions. This choice is not restrictive and any other interval could be considered.

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Following the approach of kinetic theory, the proposed model relies on the definition of a function $f(v,t)$ which represents the density of opinion v at time t and which is defined for each opinion $v \in I$ and for each time $t \geq 0$. Since $f(v,t)$ is a density function, the integral over I of $f(v,t)$ with respect to v equals the number of agents in the considered multi-agent system at time t, which is denoted as $n(t)$. In order to describe the opinion evolution using a kinetic approach, we assume that the function $f(v,t)$ evolves according to the Boltzmann equation. In particular, we consider the following (homogeneus) formulation of the Boltzmann equation

$$\frac{\partial f}{\partial t} = \mathcal{Q}(f,f)(v,t) \qquad (1)$$

where the left-hand side represents the temporal evolution of the distribution function and $\mathcal{Q}$ is known as *collisional operator*, since it takes into account the effects of interactions. The explicit expression of the collisional operator is strongly related to the microscopic rules used to describe the effects of interactions among agents. The following model has been considered in the literature, according to which the post-interaction opinions of two interacting agents are obtained from the formulas

$$\begin{cases} v' = v + \gamma(w-v) + \eta_* D(|v|) \\ w' = w + \gamma(v-w) + \eta D(|w|) \end{cases} \qquad (2)$$

where: (v', w') denote the post-interaction opinions of the two agents whose pre-interaction opinions were (v,w); γ is a deterministic parameter in $I_\gamma = (0, \frac{1}{2})$; η and η_* are two random variables; and $D(\cdot)$ is known as diffusion function. In this case, it is possible to prove that the number of agents n and the average opinion u of the systems are constant parameters [7]. Stationary profiles g for opinion distribution can be found as solutions of

$$\frac{\lambda}{2}\frac{\partial^2}{\partial v^2}(D(|v|)^2 g) + \frac{\partial}{\partial v}((v-u)g) = 0 \qquad (3)$$

and they depend on the explicit expression of the diffusion function $D(\cdot)$. We remark that if $D(\cdot) = 0$, then consensus is reached.

More recently, the kinetic approach has been extended to the study of opinion formation in multi-agent systems with different classes of agents [2]. In the following, we assume that M classes of agents coexist inside the same system and we denote as n_s the number of agents of class s and as $u_s(t)$ the average opinion of agents of class s. We assume that the post-interaction opinions of the two interacting agents depend on the pre-interaction opinions according to the following rules

$$\begin{cases} v' = v - \gamma_{sr}(v-w) \\ w' = w - \gamma_{rs}(w-v) \end{cases} \qquad (4)$$

where v, w, v' and w' have the same meaning as in (2) and $\{\gamma_{sr}\}_{s,r=1}^{m}$ are deterministic parameters defined in I_γ. The value of γ_{sr} measures the propensity of an agent of class s to change its opinion in favor of the opinion of an agent of class r. In this case, a proper differential equation for each class needs to be defined, namely

$$\frac{\partial f_s}{\partial t}(\underline{v}, t) = \sum_{r=1}^{m} \mathcal{Q}_{sr}(f_s, f_r) \qquad \forall\, s \in \{1, \dots, M\}. \qquad (5)$$

Under these assumptions, it can be proved that the number of agents of each class is conserved and that the asymptotic average opinions of all classes are equal. Under these assumptions, it can be proved that the number of agents of each class is conserved and that the asymptotic average opinions of all classes are equal.

3 Conclusion and Ongoing Work

In this paper, we discussed analytic models for the study of opinion dynamics and consensus formation in multi-agent systems. The considered models are inspired from a framework related to kinetic theory of gases and of gas mixtures. Kinetic theory is typically used to derive macroscopic properties of gases starting from the description of the effects of microscopic collisions among molecules. Similarly, by reinterpreting molecules as agents and collisions among molecules as interactions among agents, one can adopt the same framework to study opinion dynamics in multi-agent systems. In particular, the adoption of kinetic theory of gas mixtures allows modeling multi-agent systems composed of various classes of agents, where each class is associated with different features. We showed that the proposed kinetic approach is sufficiently general to incorporate different models leading to different stationary profiles for the opinion.

Ongoing work involves the modeling of different microscopic phenomena, considering different rules to update the opinions of two interacting agents.

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