Towards a System-Theoretic Model for Transition of Affect

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Abstract. This study investigates the dynamic properties of affect from a system-theoretic point of view. A psychological study was conducted, in which affect was induced sequentially using pictures of the International Affective Picture System. Based on this experimental data, a piecewise linear model is formulated which describes average affective reactions of a human. System-theoretic analysis of this model reveals that the system shows complex dynamic characteristics, which can not only be explained by an additive influence of the stimulus on the current affective state. It suggests that there exist internal fluctuations. Furthermore, the joyous region contains a stable attractor. A second model concentrates on individual differences of affective reactions between humans. A Markov Chain estimates the probability that a person feels a specific affect depending on an external stimulus and the previous affective state. Using this model estimates on how a sequence of different affective stimuli influences the affective state can be calculated.
The study proves that a system-theoretic approach is suitable for modeling emotions, in particular affective states, and can give additional insights on the dynamics of emotions.

1 INTRODUCTION

Emotions influence cognitive processes, like decision making, learning, recognition and motivation. Studies analyze the source of emotions, classification of emotions and how emotions relate to cognitive processes[23, 19, 7]. Recent research investigates the integration of emotions in Human-Computer Interaction (HCI). It is presumed that recognition of human emotions, modeling emotional reactions as well as interactions, and also expressing synthesized emotions on a virtual avatar or robotic head enhances HCI [21, 20]. This requires emotion recognition, expression and dynamic modelling of emotions. A dynamic emotion model offers the possibility to predict emotional behavior, to enhance recognition systems by previous knowledge about temporal development of emotions and gives insights in complex emotional reactions. Furthermore, implementation of such a model on an emotion-expressive virtual avatar or robotic head has the benefit of compact programming compared to state machines for showing complex and versatile behavior.
This study focuses on the temporal characteristics of affect. From a system-theoretic point of view, it analyzes how the previous affective state and an affective stimulus, corresponding to external events, effect the current affective state. A time-discrete model is developed based on experimental data. As there usually exists a large variation of responses to emotional stimuli between humans, one model considers the average affective reaction of a human and analyzes the average behavior with system-theoretic tools. A second model gives estimates of how an individual would react to a specific stimulus in respect to the current affective state.
This paper is structured as follows. Section 2 gives a brief overview of related work. Then discrete affective states are defined for our purpose in section 3. Section 4 describes the psychological experiment conducted to estimate the parameters for each model. Section 5 introduces a model based on a piecewise linear system (PL), which models average affective reactions. The parameters of the model are adapted to the experimental data, followed by a system-theoretic analysis. It is concluded that internal fluctuations in affective reactions exist. Section 6 uses a Markov Chain to estimate the next affective state of a person depending on the previous affective state and an external stimulus. The paper concludes with plans for future work.

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2 RELATED WORK

Probabilistic models are currently investigated to model HCI for prediction the next emotional state of a human, analysis of the interaction and design of the affective reaction of virtual avatars or robots. Gockley at al. propose a model which is capable to calcluate synthesized social reactions based on mood, emotion and attitude for a roboceptionist [9]. Their main goal is to model long-term effects to enhance long-term relationships beteen human and robots.
Cattinelli et al. [4] consider an interaction model based on a probabilistic finite state automaton. The automaton contains a finite set of states for the robot and the user. They conclude that the interaction is stable if both agents have similar personalities. If personalities differ no quick convergence of emotional state is exprected.
Hiddden Markov Models (HMM) can be used to connect actual felt emotion with emotion expression. In [11], a HMM is proposed as a emotional core for a robot. Blewitt et al. present a computational model for defining the emotional state of an agent based on a fuzzy logic system [3, 2]. Further approaches regarding modeling emotional behavior of virtual agents or robots can be found in [1, 24, 14, 8, 5]
In this study, we focus on a model for transition of affect itself and analyze it with system-theoretic tools.

3 ASPECTS OF MODELING AFFECT

3.1 Affective States

Various theories exist to categorize emotions and affect. According to Lazarus, an emotion itself contains physiological disturbance, action tendencies that are not necessarily acted out, and affect, whereas affect is the subjective experience of a person during an emotion [13].

Most common theories are the dimensional PAD model [16], the basic emotions [6] and the primary emotions [22]. The PAD model spans a 3-dimensional space with the independent and bipolar axes valence, arousal and dominance. The current emotion of each individual can be described as a state within this space. The basic emotions are anger, disgust, fear, joy, sadness, and surprise. They are derived from facial expressions common among all cultures. In contrast to the basic emotions, the primary emotions can be blent together, so that each emotion consists to a certain part of the primaries which are acceptance, anger, anticipation, disgust, joy, fear, sadness and surprise.

In this work, we combine the dimensions arousal and valence with the discrete emotions sad, joyous, neutral and quiet, which are derived from the basic emotions, see Fig. 1 . Mikels et al. collected descriptive emotional categories for IAPS to identify images which elicit one discrete emotion more than others. Based on their work, the discrete state sad represents the area in the arousal-valence space with which participants most associated the state sad in Mikels et al.'s study.

Descriptive emotional categories as awe, content, and amusement are mixed for pictures in the upper plane of the arousal-valence space [17]. We choose the area where arousal values range from 4 to 6 and valence values range from 6 to 8 for the state joy.

Morris scored advertisements and emotion adjectives using SAM in his study. The number of pictures available in the IAPS data set shows a V-structure with its peak at medium pleasure and low arousal. In Morris studies, adjectives like quiet and solemn correspond to this area. We defined this area as the discrete state quiet.

The discrete state neutral represents the area between sad and joyous. For equality of the states, each region spans a two times two square in the arousal-valence state space. Furthermore, the states sad, neutral and joyous have the same range of arousal, so that state transitions depending only on pleasure can be investigated.

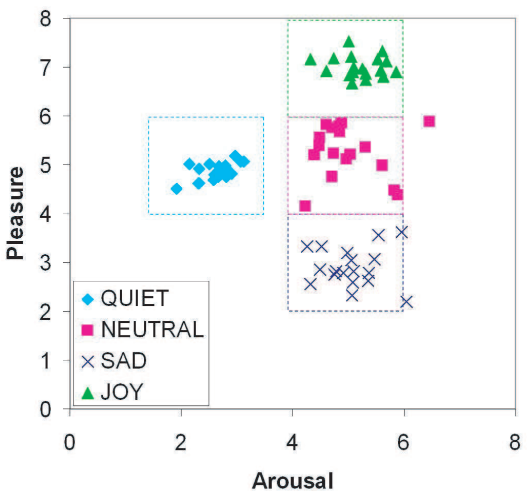

Figure 1. The four discrete emotional states quiet, sad, joyous and neutral are arranged in the two-dimensional arousal-valence subspace of the PAD model. The markers display the affective states of the IAPS pictures used for induction of a specific affect.

3.2 System-theoretic Model

There exists a large variety of system-theoretic models. In general, they can be divided in stochastic and deterministic models. This study uses a Markov Chain as a representative for a stochastic approach and a Piecewise Linear System for deterministic modeling. A data-driven concept is chosen to determine the properties for each model, as there is no physical law applicable to emotions in order to derive state equations. A psychological experiment is conducted for this purpose. The experiment is repeated over a number of participants, so that a model can be formulated which describes the mean affective behavior. This model specifies the average affective reaction of a human. The analysis of this model concentrates on the effect of the previous affective state on the current affect in comparison to the affective stimulus. Furthermore, it investigates if in general more stable or instable affective regions exist.

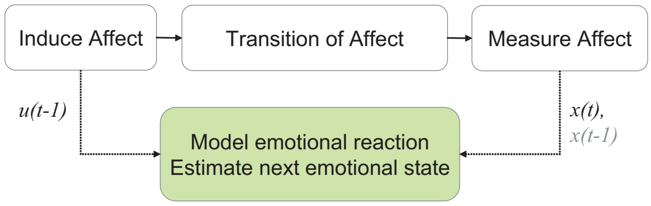

Figure 2. A stochastic as well as a deterministic model are designed based on knowledge about the affective stimulus $u(t-1)$ and the actual affective state $x(t)$ of a person. The previous affective state $x(t-1)$ is estimated based on ratings in the IAPS manual.

However, emotional reactions vary within subjects and in-between subjects. This leads to a Markov Chain which estimates the probability that an individual person feels a specific affect. This model gives estimates for each affective state considering the previous emotional state $x(t-1)$ and the stimulus $u(t-1)$.

4 PSYCHOLOGICAL EXPERIMENT

A psychological experiment is contucted in order to identify parameters of the Piecewise Linear System and the Markov Chain. It is essential to set the scene for interpretation and system-theoretic analysis of each model.

4.1 Preparation of Emotion Induction

IAPS was choosen for emotion induction because it is continously updated and evaluated. The IAPS pictures are sorted by their valence, arousal and dominance values. 18 pictures for each emotional state were selected from the IAPS database. Each picture was shown only once. Criterion for selection was the standard deviation of arousal, valence and dominance, which ranges between 1 and 3 in the data base. In the three subsets for sad, quiet and neutral, pictures were selected with a standard deviation lower than 2 for arousal and valence. Pictures which induce joyous vary more in their standard deviation so that the maximum standard deviation for arousal and valence is 2.14.

The pictures were aligned in a presentation so that each transition between the four states is repeated three times. It was achieved to keep arousal almost constant for transition between the states sad, neutral and joyous. This allows independent investigation of transitions along the pleasure axis.

Due to different reactions of humans to erotic pictures, they were excluded.

4.2 Test Procedure

The presentation which includes the IAPS pictures was shown with a video projector. The slide transitions were determined by a timer. Each picture was shown 5 seconds. The participants had 15 seconds time to fill out the Self Assessment Mannequin (SAM) questionaire. After that a start slide was shown with the number of the next transition for 5 seconds.
A maximum of 8 persons completed the experiment at the same time. Overall 50 persons participated at the experiment.
The handout for the participants contained a how-to-do manual for SAM, test questionaires, the SAM questionaires for the transitions and an EPI personality test at the end.
The general procedure of the experiment was as follows

- Initial hellos
- Supervisor explains the experiment procedure and reads out the manual for the SAM
- Time for questions
- 1 test cycle (IAPS picture + SAM evaluation)
- 1 IAPS pictures of each state to identify a participant's peculiar response to emotion induction
- 3*3*4 test sequences (3 transitions for each state, 3 repetitions, 4 states)
- EPI personality questionaire.

One procedure took about 30 minutes.

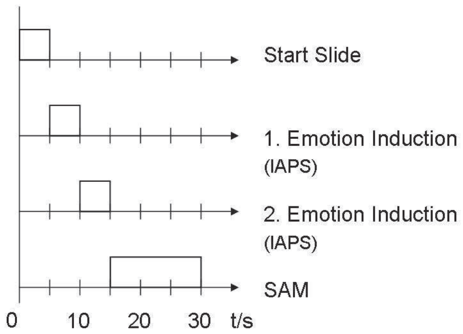

Figure 3. This graph shows a single cylce to investigate the transition from one emotional state to another. First the start slide with the number of the cycle was shown. After 5 seconds the first IAPS picture was presented. The second emotion induction followed 5 seconds later and the picture was also shown for 5 seconds. Then the SAM questionaire was filled out. This cycle was repeated 36 times.

4.3 Discussion of Experimental Data

The averaged ratings of our participants over all pictures differ from the ratings listed in the IAPS manual. The mean arousal value is 1.8 lower in average to IAPS. The mean valence value is 0.8 lower in average to IAPS. This results in a shift of the discrete regions for quiet, joyous, neutral and sad in the valence-arousal space (see Fig. 6) . The

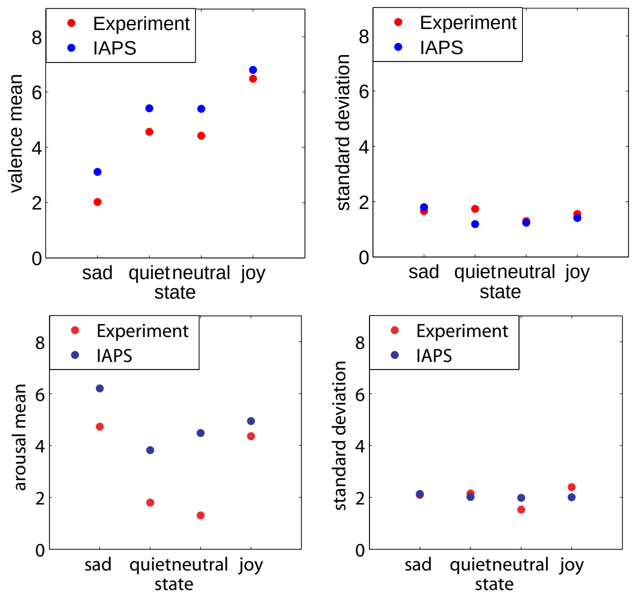

Figure 4. The ratings for valence and arousal are lower in our experiment comparing to the lists in the IAPS manual. The standard deviation is almost equal.

mean valence values over all participants are plotted separately for each affective state in Fig. 5.The degree of valence is usually higher for the second affective state if the previous state was quiet or neutral. If the first affective state was sad, participants rated in average their next emotional state with less valence. In average, the discrete induced affect for the second state was always reached, nevertheless what the previous affective state was. The averaged arousal values were also within the expected discrete states. It is aimed that the following state model is capable to model slight differences in emotional reaction to a stimulus depending on the previous state.

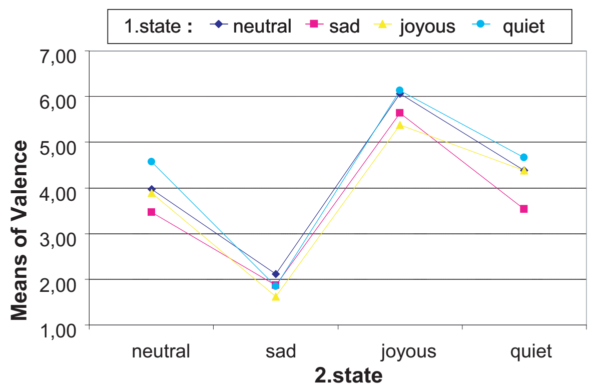

Figure 5. This plot shows the mean ratings of valence depending on the previous affective state. In average, participants rated valence higher if the previous affective state was neutral or quiet.

5 MODELING THE MEAN TRANSITION OF AFFECT: STATE MODEL

A time-discrete, time-invariant, Piecewise Linear System (PL) with state-depented switching is proposed to model transition from one affective state to another [15, 10]. This model covers the average behavior over all participants. As it is supposed that affective reactions are complex, dynamical and nonlinear, a set of linear dynamic equations are chosen for approximation.
The state vector X_k consists of two dimensions arousal and valence. It is measured with the SAM rating during the experiment. As the SAM test ranges between 1 and 9, the components of X_k also range continuously between 1 and 9.

$$X_k = \{valence, arousal\}^T \tag{1}$$

The influence of emotion induction is modeled by the input vector u_k. It reflects the difference of the expected PAD values of 2 consecutive IAPS pictures. Furthermore, the influence of the emotion induction u_k is delayed and affects the state X_{k-1}.

$$u_k = \Delta u = \{\Delta valence, \Delta arousal\}^T \tag{2}$$

The output vector Y_k is a unit vector e_q of $S_Y = \{e_{joy}, e_{neutral}, e_{sad}, e_{quiet}\}$. Each unit vector e_q represents one discrete affective state sad, neutral, quiet or joyous.
The state space for arousal and valence is divided in four regions according to Fig. 6. The behavior in each region is approximated by the following linear dynamic equations:

$$\begin{aligned} X_{k+1} &= A_q X_k + B_q u_k + X_q && (3)\\ Y_k &= C X_k + Y_0 + w_k && (4)\\ q &\in \{sad, joyous, neutral, quiet\} \end{aligned}$$

The probability matrix A_q describes internal fluctuations in human emotions. Even if no external stimuli is present, the emotion will drift. The impact of the external stimuli u_k is defined by the matrix B_q.
The mapping between the continuous state X_k and the discrete affective output Y_k is realized by the matrix C. This mapping can be derived from our definition of the discrete outputs. The martingale w_k is required so that Y_K reaches the most probable output calculated by CX_k (4). The value of the martingale increment is adapted for each time step k. The sample rate is 0.2 Hz.
If the present state X_k would not influence the next emotional state and if the input underlies no damping or amplification, the state equation would look as follows:

$$X_{k+1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} X_k + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} u_k + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

The matrices A_q and B_q are identity matrices for all q. If mean emotional behavior is completely covered by Equ. (5), affective reactions are barely a reaction to affective stimuli and the previous affective state does not influence the current affective state.
If this hypothesis does not hold, the conclusion can be drawn that complex dynamics underlie emotional reactions.

5.1 Estimation of the Output

The output describes the mapping between the continuous states valence and arousal to the four discrete states joyous, neutral, sad and quiet. The matrix C and the offset Y_0 need to be estimated.

$$Y_k = CX_k + Y_0 + w_k \tag{6}$$

The switching boundaries between the states are defined in Fig.6. Their mathematic description is as follows:

$$\begin{aligned} f_1 &= (1\,0)X_k - 3,2 \\ f_2 &= (1\,0)X_k - 5,2 \\ f_3 &= (0\,1)K_k - 2,2 \end{aligned} \tag{7}$$

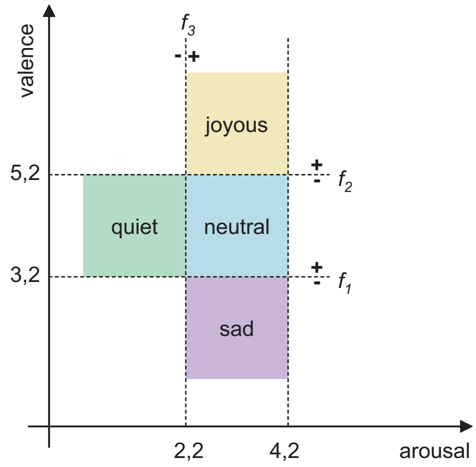

Figure 6. This graph shows the switching boundaries between the regions. The most probable discrete state Y_k is estimated by superposition of the functions f_1, f_2 and f_3 .

The signs of f_1, f_2 and f_3 differ depending on the discrete states, e.g. for a specific state in the region for neutral, f_1 and f_3 are positive and f_2 is negative.
The first element of the vector calculated by $CX_k + Y_0$ is asso-

[htbp]

Table 1. Values of functions in discrete regions

region	f_1	f_2	f_3
joy	+	+	+
neutral	+	-	+
sad	-	-	+
quiet	+	-	-

ciated with the discrete state joy, the second with neutral, the third with sad and the last with quiet. So that the first row represents the superposition of the functions for the state joy in that way, that the resulting function is maximum if the state is in the region of joy otherwise less (according to table 1). This procedure is analog for the following three rows of Y_k. The following equation combines the superposition of the decision functions for the four discrete states.

$$Y_k = \begin{pmatrix} 2 & 1 \\ 0 & 1 \\ -2 & 1 \\ 0 & -1 \end{pmatrix} X_k + \begin{pmatrix} -10,6 \\ -0,2 \\ 6,2 \\ 4,2 \end{pmatrix} + w_k \tag{8}$$

Usually, probabilities range between 0 and 1. The above equations are shift to positive values for valence ranging between 0 and 9, and arousal ranging between 0 and 9, so that the result is positive for all possible SAM ratings. Also, they are normalized to guarantee a

maximum value of 1 and a minimum value of 0.
The martingale w_k is required so that Y_k reaches the most probable state and can be represented by unit vectors.

$$Y_k = \frac{1}{28,2}\begin{pmatrix} 2 & 1 \\ 0 & 1 \\ -2 & 1 \\ 0 & -1 \end{pmatrix} X_k + \begin{pmatrix} 0.0426 \\ 0.4113 \\ 0.6383 \\ 0.5674 \end{pmatrix} + w_k \tag{9}$$

5.2 Estimation of State Equation

Actually, one state equation was considered. However, it is not possible to reflect complex behavior within one linear equation. So the state equation were extended to a PL system. A_q, B_q, and X_q are adapted separately for each state.
The average arousal and valence values over all participants were calculated for each experimental cycle. They are the values for X_{k+1} and represent the emotional state of a participant after the second stimulus. The emotional state after the first stimuli was not measured and is derived from the values in the IAPS manual. Besides X_{k+1} and X_k, u_k is known. It is the difference in arousal and valence between the first and the second stimulus.
Least Square was used for estimation of the parameters. Exemplarily, the calculation of $a_{11}, a_{12}, b_{11}, b_{12}$ and $X_{0,v}$ is shown for the discrete state joyous. These parameters are required to estimate the valence component of X_{k+1}.

$$\begin{aligned} X_{k+1,v} &= a_{11}X_{k,v} + a_{12}X_{k,a} \\ &+ b_{11}u_{k,v} + b_{12}u_{k,a} + +X_{0,v} \end{aligned} \tag{10}$$

The unknown parameters are combined in the vector $x = [a_{11}, a_{12}, b_{11}, b_{12}, X_{0,v}]^T$. Each transition was repeated three times. Also, if the first emotional state is joyous, the next state can be sad, neutral or quiet. In total, we have nine times measured state variables and input variables for the above equation. So that a data matrix D is introduced, whose rows j contain the input and state values for each of the nine transitions.

$$D_j = [X_{k,v,j}\ X_{k,a,j}\ u_{k,v,j}\ u_{k,a,j}\ 1] \tag{11}$$
$$\text{with } j \in 1...9$$
$$d = [X_{k+1,v,1}...X_{k+1,v,9}]^T \tag{12}$$

The next states $X_{k+1,v,j}$ are combined in the vector d. Least Square is used to solve the overdetermined system

$$d = Cx. \tag{13}$$

This procedure was repeated for arousal, which is the second component of X and for each region. This results in the following state equations.
State equation for the region joy:

$$X_{k+1} = \begin{pmatrix} 0.4 & -0.6 \\ 0.7 & -0.4 \end{pmatrix} X_k + \\ + \begin{pmatrix} 0.9 & -0.4 \\ -0.3 & 0.8 \end{pmatrix} u_k + \begin{pmatrix} 5.2 \\ 0.2 \end{pmatrix} \tag{14}$$

State equation for the region neutral :

$$X_{k+1} = \begin{pmatrix} 1.0 & -1.2 \\ 0 & 1.0 \end{pmatrix} X_k + \\ + \begin{pmatrix} 1.3 & -0.3 \\ -0.3 & 1.0 \end{pmatrix} u_k + \begin{pmatrix} 3.6 \\ 0.4 \end{pmatrix} \tag{15}$$

State equation for the region sad:

$$X_{k+1} = \begin{pmatrix} 1.6 & -0.8 \\ -0.6 & 1.3 \end{pmatrix} X_k + \\ + \begin{pmatrix} 1.0 & -0.3 \\ -0.1 & 0.8 \end{pmatrix} u_k + \begin{pmatrix} 0.5 \\ 0.7 \end{pmatrix} \tag{16}$$

State equation for the region quiet:

$$X_{k+1} = \begin{pmatrix} 1.3 & 1.1 \\ -0.8 & 0.8 \end{pmatrix} X_k + \\ + \begin{pmatrix} 1.1 & 0.0 \\ -0.2 & 0.6 \end{pmatrix} u_k + \begin{pmatrix} -2.4 \\ 4.8 \end{pmatrix} \tag{17}$$

There are large variations in the parameters of A_q for each region. This proofs that an individual model for each region is required. The elements of B_q are similar. As expected, the elements on the diagonal are larger than the elements on the secondary diagonal so that the component arousal of the stimulus influences arousal more than valence of the state and vice versa.
So far, this model covers only the average emotional behavior of humans.

5.3 Evaluation

5.3.1 *Evaluation of A_i, B_i*

The root mean-squared error e_j is calculated between the measured values $\overline{X}_{j,l}$ over all participants for each transition l and the simulated value X. The errore_j is calculated for each discrete state j separately.
It is averaged over the three repetitions of each transition and over all transition from one discrete state.

$$\begin{aligned} e_j &= \sqrt{\sum_{l=1}^{n}(\overline{X}_{j,l} - X_{j,l})^2/(n-1)} \\ &= \sqrt{\sum_{l=1}^{3}(\Delta X_{j,l})^2 \cdot 0.5} \end{aligned} \tag{18}$$

The following table shows the mean quadratic error for valence (v) and arousal (a) for each transition:

Table 2. Root mean-squared error of simulation

1.state	v/a	sad	2.state joy	neutral	quiet	overall
sad	v	\	0.9	0.1	0.3	0.5
	a	\	0.5	0.4	0.3	0.4
joy	v	0.5	\	0.2	0.2	0.3
	a	0.5	\	0.5	0.6	0.4
neutral	v	0.1	0.3	\	0.2	0.2
	a	0.3	0.1	\	0.2	0.2
quiet	v	0.4	0.4	0.4	\	0.3
	a	0.3	0.4	0.6	\	0.4

The error alternates between 0.2 and 0.5. However, emotion induction has a large variation (standard deviation between 1 and 2), so

that the simulation errors are minor comparing to variations caused by individual emotional responses or reactions.
The total root mean-squared error is 0.32 for valence and 0.35 for arousal.

5.3.2 *Evaluation of State Order*

Fig. 5 shows that participants felt more valence, if the previous state was neutral or quiet. The same plot was generated by the PL system. Comparison of both plots showed that the model predicts the same behavior for joyous, neutral and sad as the second state. If the second state is quiet, the mean values for a neutral and joyous previous state lie close to each other. In the simulation, their order is vice versa. Same results are gathered for analyzing the mean values of arousal.

5.3.3 *Reachability of the Discrete States*

In average, all participants reached the induced emotional states for all transitions. The same holds for simulation of all transitions.

5.4 Systemtheoretic Analysis

Methods from control and system theory are applied to the obtained state equations, so that conclusions can be drawn about stability, observabilty and detectability for each affective region. This analysis interprets the mean affective behavior over all participants. The external stimulus u_k is set to zero. This means that the external stimulus stays constant, e.g. in case of our experiment showing the same joyous or sad IAPS picture for a longer time period and try to induce the same affective state for a longer time period. The reaction differs depending on the affective state at the beginning.

5.4.1 *Stability*

First, the fixed point for each affective region is calculated using the corresponding linear dynamic equation. The fixed points of the dynamic equations for joyous and sad lie within their regions, the others outside. Table 3 lists the coordinates for each fixed point.

Table 3. Fixed points

region q	valence	arousal
sad	2.6	2.7
joy	5.4	2.9
neutral	12.3	2.9
quiet	5.8	0.5

As the system is time-discrete, the absolute value of the eigenvalues must be lower than one for a stable fixed point. Applying Lyapunov's Indirect Method to each fixed point, yields that the system has one stable fixed point, one instable fixed point and two saddle points (see Table 4).
The phase diagram of the PL system is plotted in Fig.7. However, this diagram is plotted for $u_k = 0$. This case, repeating the same stimulus, was not included in the experiment, so that the state equations are only approximated by data for alternating stimuli. Interpreting the phase plot leads to assumptions which need to be checked in a second experiment which includes repeating the same stimulus. For this model, the following interpretations are drawn. If a joyous picture

Table 4. Eigenvalues of the state matrix A_q

region q	1.Eigenvalue	2.Eigenvalue	
sad	2.1	0.8	saddle point
joy	$0.5i$	$-0.5i$	stable fixpoint
neutral	0.8	1.2	saddle point
quiet	$1.0 + 0.9i$	$1.0 - 0.9i$	instable fixpoint

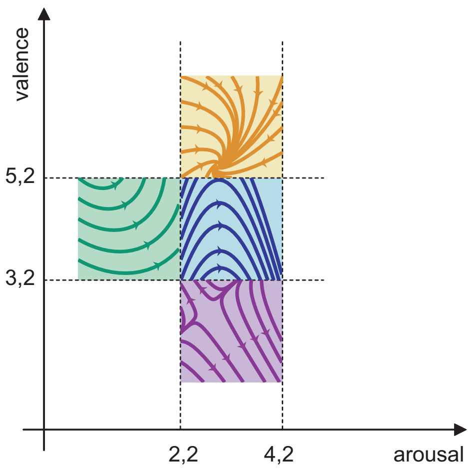

Figure 7. This graph shows the phase diagramm for each region for $u_k = 0$

is shown repetitively, the mean reaction over all participants would be, that a stable joyous affect is reached. However, a sad stimulus with less pleasure and medium arousal would lead to more sadness and eventually anger. Starting with a quiet or neutral affect, arousal increases over time.

5.4.2 *Controllability*

Kalman method is used to analyze controllability of the PL system for each region. For all regions, the rank of the controllability matrix $Q_{B,q} = [B_q, A_q B_q]$ was equal the number of states, so that each region is controllable.

5.4.3 *Observability*

In analogy, observability is studied for each region. The observability matrix $Q_{O,q} = [C^T, A_q^T C^T]$ is calculated. As the rank for all observability matrices $Q_{O,q}$ is equal the number of states, all regions are observable.

6 MODEL OF AFFECTIVE TRANSITIONS FOR ONE INDIVIDUAL: MARKOV CHAIN

A Finite Markov Chain is considered to model individual fluctuations from mean affective behavior [12]. This model calculates the probability that a specific person feels a specific affect influenced by an external stimulus. The affective state and the stimulus can take four discrete values joyous, neutral, quiet and sad. The probability vector p_k at a specific time step k contains the probability for each affective state.

$$p_k = (p_{joy}, p_{neutral}, p_{sad}, p_{quiet}) \tag{19}$$

The parameters of the probability matrix $\Omega_{stimulus}$ depend on the external stimulus.

$$\Omega_{stimulus} \in \{\Omega_{joyous}, \Omega_{neutral}, \Omega_{sad}, \Omega_{quiet}\} \tag{20}$$

The probability that a specific person feels a specific affect depends only on the external stimulus and on the previous affective state.

$$p_{k+1} = p_k \Omega_{stimulus} \tag{21}$$

The probability matrices differ for each type of stimulus. Table 5 to 8 lists the matrices for joyous, neutral, sad and quiet stimuli separately. As the same stimulus was not repeated in this experiment, the according row is filled with equally distributed probabilities. These probabilities need to be estimated in a continuative experiment. The interpretation of the tables is as follows. If the previous state is quiet and the external stimulus is joyous, see Table 5, the probability that a person feels joyous is 74%. However if the previous state is sad, the probability is only 36% that a person reacts to a joyous stimulus with joy. This model can be used to simulate the effect of a sequence of stimuli. Alternating a sad and a joyous stimulus leads to a higher probability that a person feels sad. So it can be concluded that the sad stimulus has a stronger effect.

Table 5. Probability matrix for joyous stimulus

	predicted state			
previous state	joyous	neutral	sad	quiet
joyous	0.25	0.25	0.25	0.25
neutral	0.68	0.12	0	0.2
sad	0.36	0.16	0.14	0.34
quiet	0.74	0.02	0.04	0.2

Table 6. Probability matrix for neutral stimulus

	predicted state			
previous state	joyous	neutral	sad	quiet
joyous	0.2	0.14	0.44	0.14
neutral	0.25	0.25	0.25	0.25
sad	0.06	0.16	0.52	0.26
quiet	0.5	0.08	0.14	0.28

Table 7. Probability matrix for sad stimulus

	predicted state			
previous state	joyous	neutral	sad	quiet
joyous	0.02	0.04	0.88	0.06
neutral	0.1	0.06	0.64	0.2
sad	0.25	0.25	0.25	0.25
quiet	0	0.02	0.94	0.04

Table 8. Probability matrix for quiet stimulus

	predicted state			
previous state	joyous	neutral	sad	quiet
joyous	0.3	0.06	0.12	0.52
neutral	0.12	0.1	0.08	0.7
sad	0.04	0.12	0.44	0.4
quiet	0.25	0.25	0.25	0.25

7 DISCUSSION

The first model based on a piecewise linear system analyzes average affective transitions of a human. The linear approximations in each region joyous, neutral, quiet and sad differ from equation 5. This concludes that superposition of external stimulus and affective state is not sufficient to explain affective transitions. Furthermore, the states valence and arousal are controllable and observable in each region of the state space. As properties of nonlinear systems are only locally valid, in this case in each region, a reachability analysis of the complete PL system is considered in future works. Within the joyous region lies a stable fixpoint. It is assumed that repetition of joyous stimuli maintains joy. However, reactions to a repetition of sad stimuli depends on the initial value of arousal and valence. It can drift to anger or neutral. As the underlying experimental data did not contain repetition of the same stimulus, these are assumptions. A continuaitive experiment is ongoing.

A second model predicts the probability for the next affective state depending on the current and an external stimulus. A Markov Chain ist proposed with a separate probability matrix for each stimulus joyous, neutral, quiet and sad. Mostly, the probability for the next state is highest for affective state intended by the external stimulus. For neutral and quiet stimuli, the previous state influences the next stronger comparing to sad or joyous stimuli. This model can be used to calculate the probabilities for each affective state for a sequence of stimuli. For HRI interaction scenarios, this model offers the benefit, that combinations of different stimuli can be simulated with this model beforehand.

The Markov property holds for both models. For simplification, we assume that the future affective state depends only on the present emotional state and is independent of its history. Long-term effects are neglected within this study.

8 CONCLUSION AND FUTURE WORK

This study shows that system-theoretic approach is applicable to model transitions of affect. Two models are proposed. One models the average affective reaction of a human and gives insights on the dynamics of affect. Simple superposition of external stimulus and current affective state is not sufficient to predict the next affective state. It is concluded that affect underlies more complex dynamics. These dynamics are approximated by a piecewise linear system. System-theoretic analysis revealed that a stable fixed point exists within the joyous region. This suggests that joyous is a stable condition over time. However, a continuative study is required which considers repeating the same stimulus in the experiment. A second model based on a Markov Chain estimates the probability how a person reacts to an external affective stimulus. This model is capable to predict the affective reaction to a sequence of affective stimuli.

Up to this point, only transitions of affect have been considered. The models need to be extended to cover also repetition of the same external stimulus. Furthermore, personal traits, like extrovert/introvert, degree of anxiety, gender or mood, can have an influence on the affective reaction. For both reasons, a similar psychological experiment is planned. The long-term target is to test its validity in a real-world HRI setting.

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