

Exactly Solved Models for Collective Behaviour and Complex Systems

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Abstract

Complex systems is a set of theories which serves to model emergent behaviours or emergent phenomena in systems which are not reducible to the sum of their parts. Among these theories is the modelisation of collective phenomena. As miracles of regularity in nature appears the exceptional family of "exactly solved model" as the Ohm formula $U = RI$ which is so simple but after all is a relation between statistics of electrons.

This talk begins by recalling the "triple birth of quantum mechanics" and the history of ideas leading to the fundamental commutation relation $AB - BA = 1$. We show how these operators can be found in the macroscopic world as simple statistical "death and birth" processes and gain a very natural and easy description of the traditional Fock space then explaining the necessity of the normal form by simple spectral arguments.

Then comes the evolution groups and two aspects of the product formula applied to combinatorial field theories. Firstly, we remark that the case when the functions involved in the product formula have a constant term is of special interest as often these functions give rise to substitutional groups. The groups arising from the normal ordering problem of boson strings are naturally associated with explicit vector fields, or their conjugates, in the case when there is only one annihilation operator. We also consider one-parameter groups of operators when several annihilators are present. Secondly, we discuss the Feynman-type graph representation resulting from the product formula and show that there is a correspondence between the packed integer matrices of the theory of noncommutative symmetric functions and these Feynman-type graphs.

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