Symposium on Mathematical Practice and Cognition

The question of how people do mathematics is currently being investigated at many different levels. "Maverick" philosophers of mathematics have moved away from traditional approaches in which an absolute validity of mathematical knowledge is sought, towards an approach in which mathematical practice is emphasised, such as how mathematical definitions or axioms are motivated, representations changed, problems discovered and explained, and how analogies are formed between different mathematical domains. In cognitive science, researchers are investigating how people think mathematically, what conceptual structures they build, what innate mathematical knowledge they are born with, how some animals can perform elementary arithmetic, etc. Within psychology, experiments drill down into the details of the underlying cognitive mechanisms of such issues as broad as the scientific process or as specific as how we conceptualise numbers, for example, where they lie in space and which conceptualisations are cultural-specific. AI researchers now have at their disposal more techniques than ever for providing a computational perspective on the question; such as computational modelling and machine learning in situated, embodied systems. Inevitably, these will contribute to testing and extending ideas about how mathematics is done. Almost every discipline could add something to the great debate of how people do mathematics.

Despite this veritable smörgåsbord of approaches and motivations, there is little interaction between different fields. We hope that this symposium will help to bridge that gap by bringing together people to talk about their work. Talks range over historical studies of great mathematicians, informal mathematical reasoning and argumentation, and how children perceive and learn mathematics. The resulting collection of talks and papers is a delightfully mixed bag, as shown below.

From the perspective of mathematical practice, Andrew Aberdein describes various "painfully familiar" types of "proof" found in mathematics and considers them in the light of Walton *et al.*'s argumentation schemes. These proofs include stating that a proof is trivial (therefore omitted); showing a proof for one or two simple cases and then considering it to have been shown to hold in the general case; and the argument that a proof must be right because if it wasn't, someone would have noticed.

Merlin Carl and Peter Koepke present Naproche, a system for parsing and understanding mathematical proof texts using techniques from computational linguistics. Their system produces semantic representations of mathematical proofs.

John Charnley gives examples of mathematical discovery from a textbook on Mathematical Analysis and discusses how his work on combining Global Workspace Theory and combined reasoning approaches could help to answer the question of how to model these discoveries. He argues that a distributed framework in which disparate reasoning systems can be brought together in such a way as to allow, for example, multiple representations, will help to create an effective mathematical discovery package.

Martin Fischer and Peter Brugger summarise research on the relation between numbers and space (the SNARC effect) and put it in the context of embodied cognition.

In a related contribution, Martin Fischer, Marianna Riello, Bruno Giordano and Elena Rusconi report on the relationship between the SNARC (Spatial–Numerical Association of Response Codes) and SMARC (Spatial–Musical Association of Response Codes) effects and find that the spatial mapping of pitch is more pronounced than that of magnitude.

Valeria Giardino points out that producing and understanding diagrams are active processes, that a diagram is only meaningful if its creator and interpreter understands the principles of its construction and that the best way to learn the meaning of diagrams is to create them.

Albrecht Heeffer describes references to body parts, or actions involving body parts, in Medieval and Renaissance arithmetic, and discusses historical functions of body parts in mathematics, and the relevance of this to the currently prevailing practice of symbolic mathematics.

Tyler Marghetis and Rafael Núñez focus on the property of mathematics to be not just a product of cognitive processes but also an activity by investigating the gestures that accompany mathematical problem solving.

Finally, Aaron Sloman asks the question "If learning maths requires a teacher, where did the first teachers come from?", and sketches out a research programme with the aim of explaining the nature of mathematical discovery, and supporting features of Kant's philosophy of mathematics.

We are very grateful to all the support that people have shown. In particular, the programme committee put a lot of time into each paper and suggesting improvements: Andrew Aberdein, Alan Bundy, John Charnley, Simon Colton, David Corfield, Martin Fischer, Thomas Joyce, Brendan Larvor, Benedikt Löwe, Rafael Núñez, Dirk Schlimm, Aaron Sloman, Alexander Svanevik and Pedro Torres. We are also especially grateful to our invited speakers: Andrew Aberdein, Alexandre Borovik, Ivor Grattan-Guinness and Brendan Larvor. Thanks to AISB for their organisation and for hosting the event, and to the AISB chair Aladdin Ayesh. Finally, very warm thanks to everyone who has come to the symposium.

Alison Pease,
Markus Guhe,
Alan Smaill.